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# An analog multiplier utilizing the nonlinear properties of thyrite

Ball, George Franklin

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# AN ANALOG MULTIPLIER UTILIZING THE NON-LINEAR PROPERTIES OF THYRITE

GEORGE FRANKLIN BALL

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G. F. Ball

AN ANALOG MULTIPLIER UTILIZING THE

NON-LINEAR PROPERTIES OF THYRITE

by

George Franklin Ball

Lieutenant, United States Navy

Submitted in partial fulfillment
of the requirements
for the degree of

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PREFACE

During the industrial experience tour of the first quarter of 1954 the author worked at the Electronic Devices Group of the Industrial Products Division, Boeing Airplane Company, Seattle, Washington, pursuing a method of multiplying two variables that was low in cost, and fairly accurate. The Thyrite Multiplier that is herein described was developed with these considerations in mind. It was used in conjunction with the Boeing Electronic Analog Computer Model 7000, with possible future use with the Model 9000. There are many methods of obtaining the products of two variables, some of which will be briefly discussed in this article. With the thought in mind that the Thyrite Multiplier was developed for fair accuracy at low cost, the author believes this multiplier to be a practical model.

In the pursuance of this project much guidance and encouragement was given the author by Mr. R. P. Abbenhouse, Mr. R. Shepard, Mr. R. D. Reid, and Mr. B. Brockway, all of the Boeing Airplane Company. To them the author would like to express his appreciation and thanks for their support. To Mr. H. C. Vivian, of Boeing's Physical Research Unit, special acknowledgment is extended for providing an experimental prototype of this type of multiplier.

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SUMMARY

This investigation is concerned with the design and development of a function multiplier for analog computers that is of low cost and simple construction. Some of the latest types of function multipliers are reviewed and compared with this multiplier. It is shown that the Thyrite quarter-square multiplier compares very favorably with these other types with respect to size, weight, cost, simplicity of operation, frequency response, and accuracy. A step by step description of the design of circuits and performance tests with test results is given, followed by an evaluation of the completed device. The Thyrite Multiplier is capable of accuracies of better than plus or minus 4% of full scale output and has a flat frequency response to better than one kilocycle per second. No auxiliary equipment is required with the exception of the Boeing Four-Unit Sign Changer, the amplifiers of which are integral parts of this plug-in multiplier.

## SECTION I

## INTRODUCTION

### 1. Need for Electronic Analog Multipliers

In this electronic age the tempo of scientific progress has increased tremendously and with this increase has come the requirement for rapid solutions to differential equations involved in everything from the design of efficient airframes for high speed jet aircraft to process controls in chemical plants. Analog computers for solving these equations have been with us for some time, but as yet no method has been found that is completely satisfactory for rapid and accurate solutions of equations containing the product of two variables, XY. This investigation was for the purpose of finding a relatively inexpensive, fairly accurate multiplier. At present a multiplier can be obtained with an accuracy of about plus or minus 2 per cent of full scale (one volt maximum error for 50 volt full scale output) for about $300. This error is most troublesome at or near zero, and causes a large error in this region compared to the quantities involved. These multipliers take up space in computer racks and are an added drain on the computer power supplies. For accuracies much better than 2 per cent, the price is high, and the multiplier is elaborate.

### 2. Classification and Description of Multiplication Methods

There are many methods of obtaining the product of two variables. Classification of these methods of multiplication can best be done according to the basic principle employed; that is:

a. Square law (7, 8, 9, 18, 26, 28, 33, 34, 37)

b. Magnetic force on an electric current (1, 14, 16, 19, 20, 30, 31)

c. Exponential law (32)

d. Modulation of a carrier (21, 22, 24, 29)

e. Rectangular area (10, 25, 35, 36)

f. Variable-gain amplifier (5, 6)

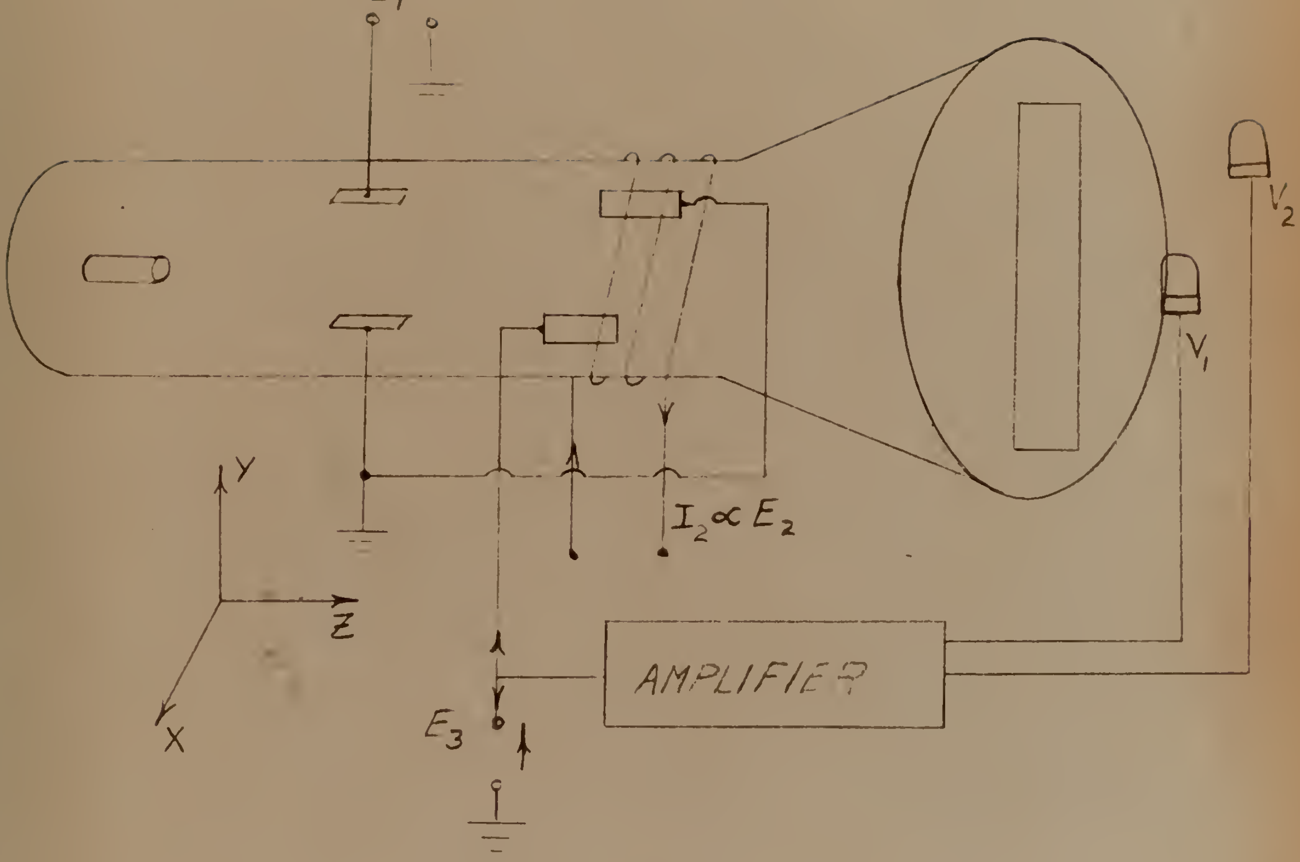
g. Variable impedance (17)

a. Square law

The square law principle is used in quarter-square multipliers. The quarter-square method is a well known way of obtaining the product of two variables. The equation illustrating this principle is:

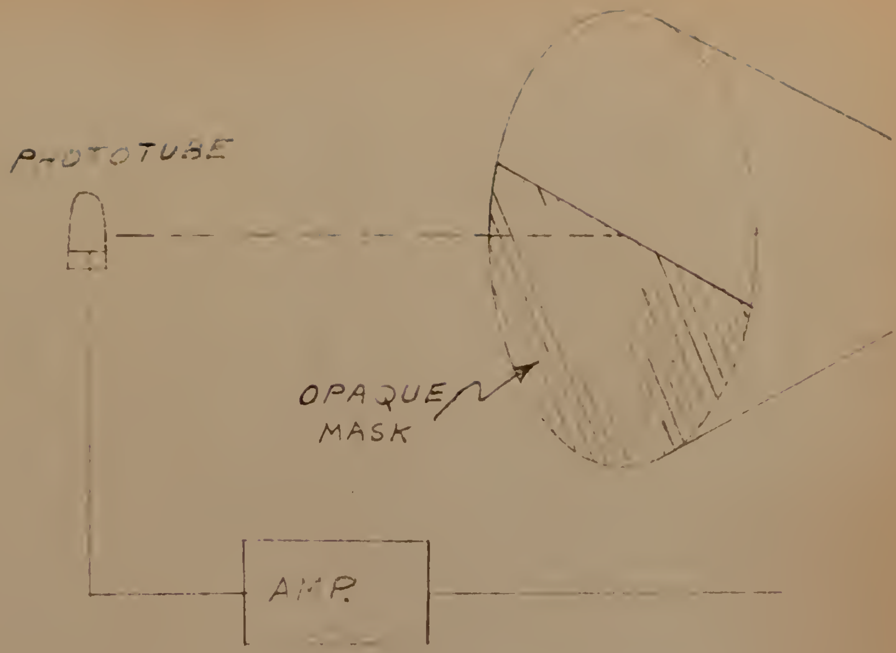
$$\frac{(A+B)^2}{4} - \frac{(A-B)^2}{4} = AB$$

Once the sum and difference of the two variables is obtained by conventional methods, a square law device is used to complete the multiplication process. One such method of obtaining a square law response is by means of a diode. The characteristic curves of diodes approximate a parabolic or square law curve. Another device is the Photoformer (8, 18, 28, 34, 37), which generates the parabolic characteristic by means of an opaque mask approximating this shape on the face of a cathode ray tube. A photo-tube forces the electron stream to follow this curve by an appropriate feedback circuit. Figure 3 shows the basic circuit of this device. A variation of this method is to make the mask on each side of the curve of differently polarized material with polarized filters on photo-tubes, such that push pull feedback circuits may

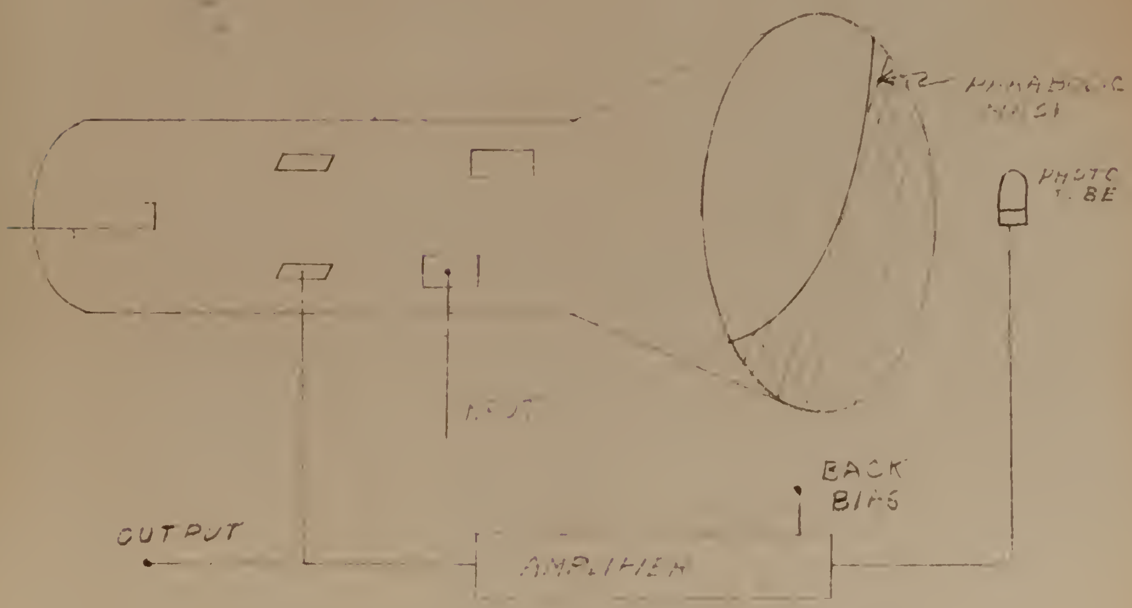



CROSSED FIELDS MULTIPLIER

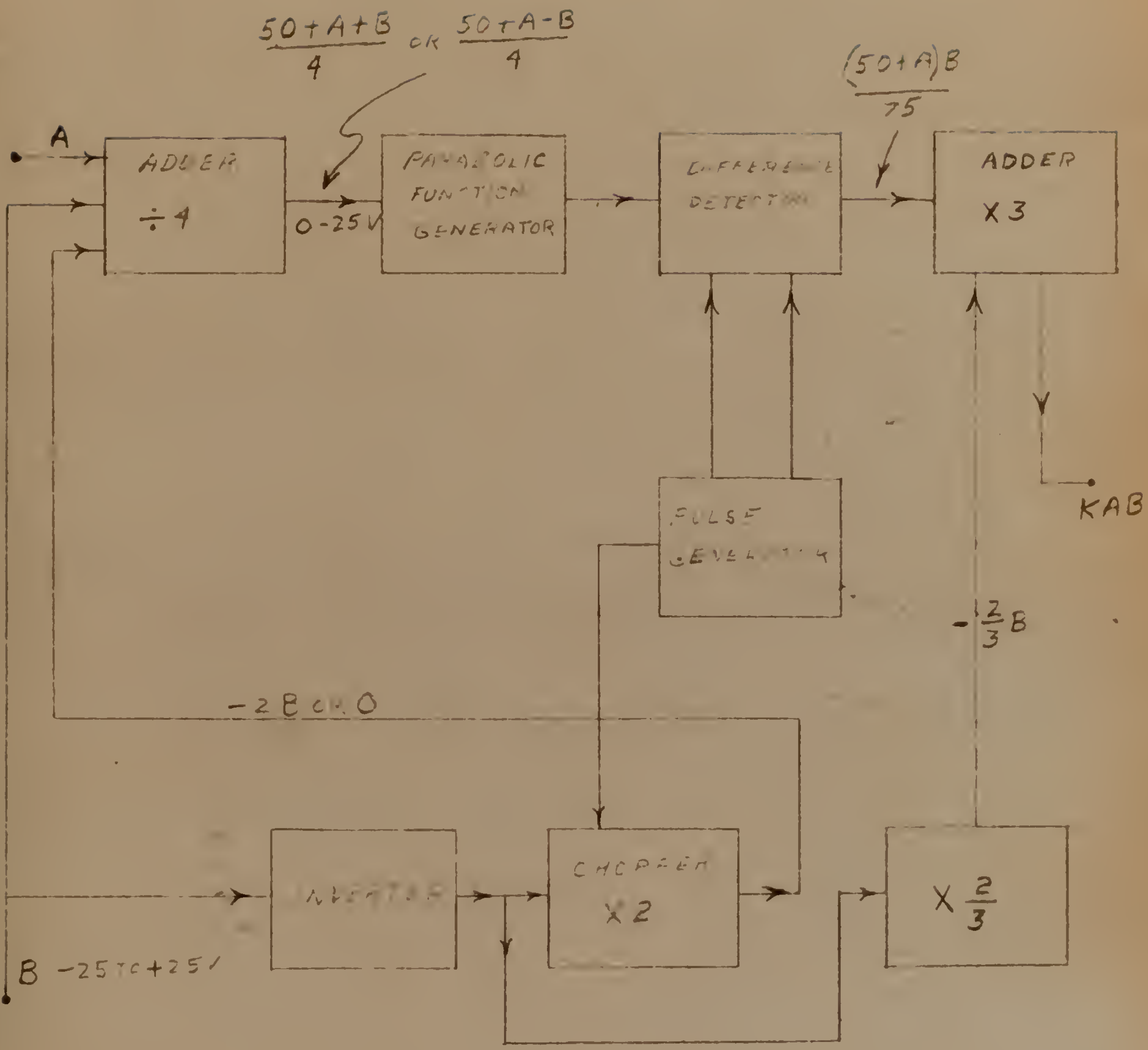
FIGURE 1




ALTERNATE FORM — CROSSED FIELDS MULTIPLIER
FIGURE 2




PHOTOELECTRIC WAVEFORM GENERATOR
FIGURE 3




QUARTER – SQUARE MULTIPLIER

FIGURE 4

be utilized.

Many other methods of obtaining square-law characteristics exist, one of which uses fifteen diodes to generate a segmented parabolic characteristic (7). A block diagram of this multiplier is shown in Figure 4. In the Thyrite Multiplier the parabolic curve is a result of the non-linearity of Thyrite.

b. Magnetic force on an electric current

The principle of using the effect of a magnetic force on an electric current to multiply takes two very different forms when actually built into circuits. The first form is that of the Crossed Fields Multiplier using a cathode ray tube (19). This is illustrated in Figure 1, and is a combination of the above principle with the effect on an electric current by an electric field. In this figure the following symbols are used:

$v_y$ - Velocity of electrons in Y direction due to electric field

$\bar{a}_x$, $\bar{a}_y$, $\bar{a}_z$ - Unit vectors in the X, Y and Z directions respectively

$B_z$ - Magnetic field in Z direction

$E_x$ - Electric field in X direction

$F_{xm}$ - Magnetic force on electrons in X direction

$F_{xe}$ - Electric force on electrons in X direction

As the electron stream passes the first pair of deflecting plates it is given a velocity

$$\bar{v}_y \propto \bar{a}_y \; E_1$$

The coaxial magnetic field coil is located at the second pair of deflecting plates, and the current $I_2$ generates $\bar{a}_z B_z$, the axial magnetic field. Then since

$$\bar{F} = e\,\bar{v} \times \bar{B}\,,$$
$$\bar{F}_{xm} = e\,(\bar{a}_y\, v_y \times \bar{a}_z\, B_z)$$

This force deflects the electron stream in the X direction. By applying a voltage $E_3$ to the second pair of deflecting plates,

$$\bar{E}_x \propto \bar{a}_x\, E_3$$
$$\bar{F}_{xe} = e\,\bar{E}_x$$

If $E_3$ is adjusted so that

$$\bar{F}_{xe} = \bar{F}_{xm}$$

then

$$\bar{E}_x = -\,(\bar{a}_y\, v_y \times \bar{a}_z\, B_z)$$

By feedback the voltage $E_3$ is adjusted automatically. Equal electrostatic and magnetic forces give zero X deflection. Photocells are placed on either side of a partition along the line of zero X deflection. The difference between the outputs is amplified and fed back to the second pair of deflection plates as the voltage $E_3$. If gain in feedback loop is great enough,

$$E_3 \propto E_1\, I_2 \propto E_1\, E_2$$

Another crossed-fields method is shown in Figure 2 (26). The Y deflection of the spot S on the screen which would normally result (corresponding to a rotation of the pattern) is counteracted by the photo-electric feedback system. The photocell prevents the spot from deviating by more than a fraction of its diameter from the edge of the

mask. The spot can thus be held closely to the X axis and $E_3$ is then proportional to the product of $E_1$ with the current $I_2$ producing B. The advantages of this method are high stability independent of the gain of the amplifier. A modification of this device is to remove the phototube and substitute a collector electrode inside the tube.

The crossed-fields method of multiplying is expensive and the accuracy depends greatly on the characteristics of the tube itself. This tube adds to the expense as it must be specially manufactured to close tolerances. Accuracies on the order of plus or minus one or two per cent of full scale can be expected.

The second type of device using the action of a magnetic field on an electric current principle is a new development in multipliers. It makes use of a phenomenon that has been known to physicists for several years, but has never before been applied to multipliers. This phenomenon is the "Hall Effect" (14, 16, 20, 30, 31). Hall Effect occurs when a material carrying an electric current is subjected to a magnetic field perpendicular to the direction of the current. A transverse potential gradient, proportional to the product of the current density in the sample and the magnetic field, is found to exist across the sample in a direction mutually perpendicular to the direction of the current and the direction of the magnetic field. The constant of proportionality is called the "Hall Constant". The Hall voltage in semiconductors is found to be quite large compared to other materials, so germanium is used in multiplier developments, because of its recent availability in greater quantities. Mathematically the Hall Effect may be stated:

Transverse potential gradient — grad $V_H = -R_H iH$

Where grad $V_H = -E_H$ (the Hall field)

$i$ = electric current density

$H$ = applied magnetic field

$R_H$ = the Hall constant

For a rectangular solid of width a and thickness b if the current distribution is uniform:

$$\text{grad } V_h = \frac{V_h}{a} = -\frac{R_h IH}{ab}$$

$$V_h = -\frac{R_h IH}{b}$$

The current consists of streams of charged particles under the influence of the electric field. With no magnetic field the streams flow longitudinally. On application of the magnetic field, the particles experience a force

$$\bar{F} \propto e(\bar{v} \times \bar{H})$$

(where v is the velocity of electron flow)

and move to the edges of the sample. The charge builds up at the edges until the electric field, due to non-uniform charge distribution, exerts a force of equal magnitude to the deflecting magnetic field. The equipotentials are rotated through an angle θ, called the "Hall Angle", determined by

$$\tan \theta = \frac{E_h}{E_x} \doteq \theta$$

(where $E_x$ is $i/\sigma$ in the direction of current flow)

$$\theta = \frac{R_h iH\sigma}{i} = R_h H \sigma$$

(where $\sigma$ is the conductivity)

(See Figure 5)

Now it can be seen that this property of semi-conductors is a very good prospect for analog multipliers, since one of the variables can be

made proportional to I and the other to H. The resulting product, multiplied by a suitable constant, is the output voltage of the semiconductor, the "Hall Voltage".

In the Boeing developmental multiplier (14) using the Hall Effect, a germanium slab is placed in the air gap of an electro-magnet as shown in Figures 5 and 6. Leads #1 and #2 are soldered to the slab and are current leads. Hall voltage leads #3 and #4 carry the output. If

$$V_h \propto e_1, \quad I \propto e_2, \quad H \propto e_3,$$

the following expressions show the applications:

For multiplication: $e_1 = c\, e_2 \cdot e_3$

For squaring: $e_2 = e_3$

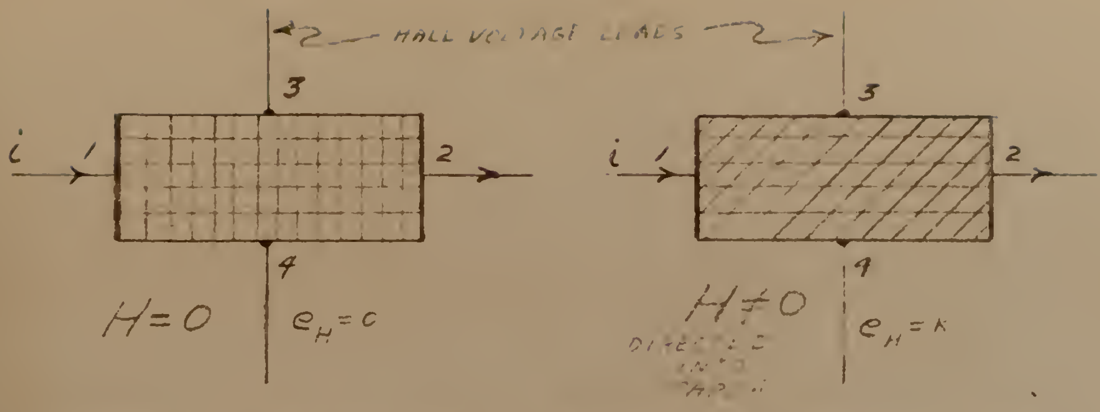
For dividing: $e_2 = \dfrac{e_1}{c\, e_3}$

For square roots: $e_2 = e_3 = \sqrt{\dfrac{e_1}{c}}$

The circuit for this multiplier is shown in Figure 7. This results in about plus or minus one per cent of full scale output as overall accuracy, and requires very complex circuitry; strict temperature compensation; and expensive materials. The need for a strong magnetic field of about 5000 Gauss calls for a large electro-magnet with resulting large inductance. Thus the multiplier is neither small nor simple, and is not any more accurate than the other multipliers discussed previously.

c. Exponential law

The exponential law principle is used in logarithmic devices (15, 32) which generate the logarithms of the two variables, add them, and extract the antilogarithm, obtaining the product in this way. Since the process is very involved, and four quadrant operation is not possible,




CONNECTIONS TO GERMANIUM STRIP SHOWING ROTATION OF EQUIPOTENTIAL LINES BY THE H FIELD

FIGURE 5

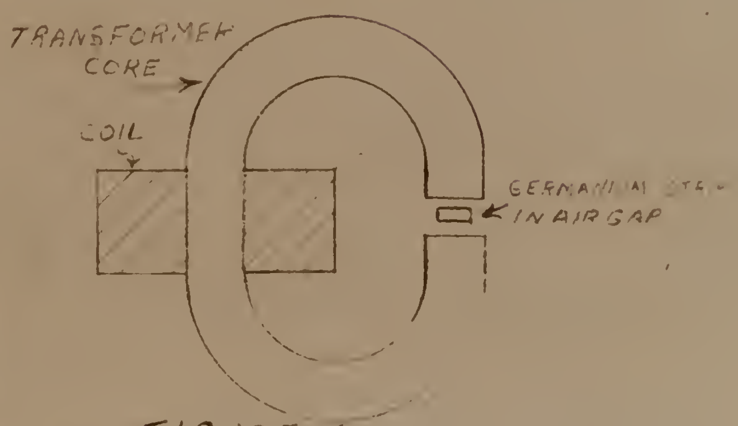
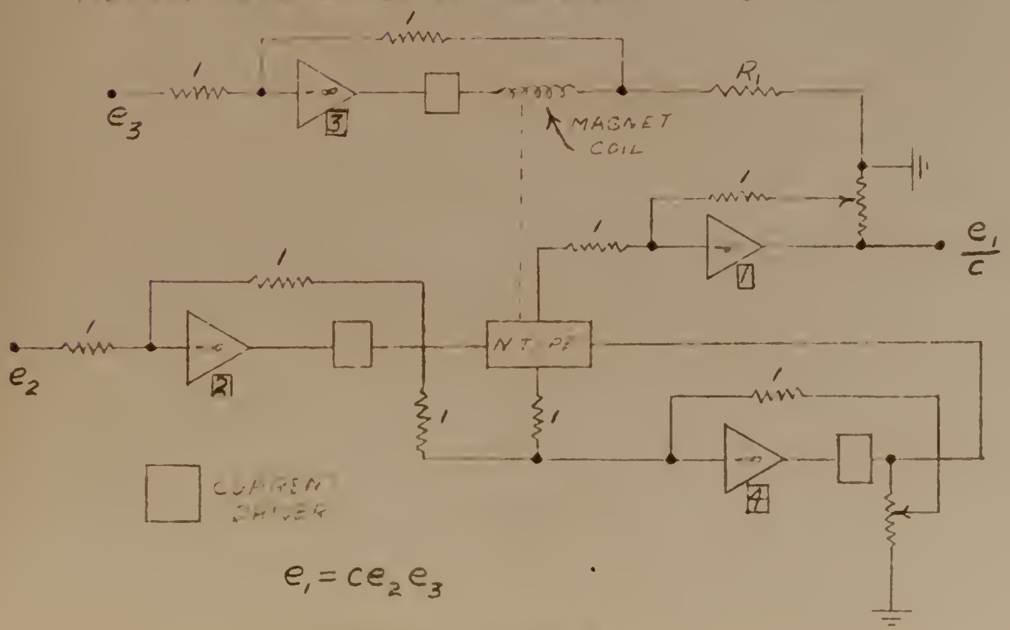



FIGURE 6

METHOD FOR APPLYING H FIELD TO STRIP




HALL EFFECT MULTIPLIER

FIGURE 7

it is doubtful if any multiplier is in use today that takes advantage of this principle. Four quadrant operation will be discussed in Section II.

d. Modulation of a carrier

Several modulation of carrier schemes are used in multiplying. One such has been developed by Price (29). It is an AM-FM multiplier with the principle of operation as shown in Figure 8. An advantage of this method is that the dynamic range at a point within the multiplier need not be the product of the dynamic ranges of the individual inputs. Another double modulation system is used by the California Institute of Technology's CIT computer (21). It is basically an AM-AM system and the block diagram appears as Figure 9. In this system the output of the first balanced modulator is of the form:

$$K_1(a_1E_1\cos w_1t+2a_2E_1E_c\cos w_ct\cos w_1t)$$

The input to the second balanced modulator is one of the sideband terms

$$K_2E_1E_c\cos(w_c+w_1)t$$

with the other sideband filtered out. The output of the second balanced modulator is of the form:

$$K_3(a_3E_2\cos w_2t+2a_4E_1E_2E_c\cos(w_c+w_1)t\cos w_2t)$$

of which the sideband term

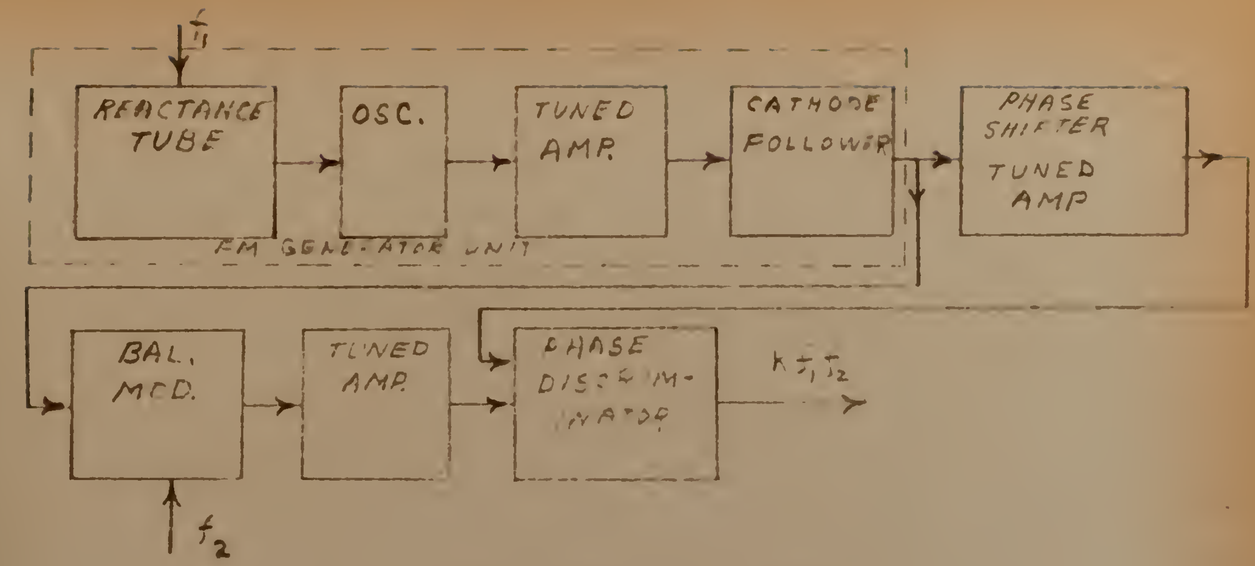
$$K_4E_1E_2E_c\cos(w_c+w_1+w_2)t$$

is the input to the third balanced modulator with the other sideband again filtered out. Its output is of the form:

$$K_5(a_5E_c\cos w_ct+2a_6E_1E_2E_c^2\cos(w_c+w_1+w_2)t\cos w_ct)$$

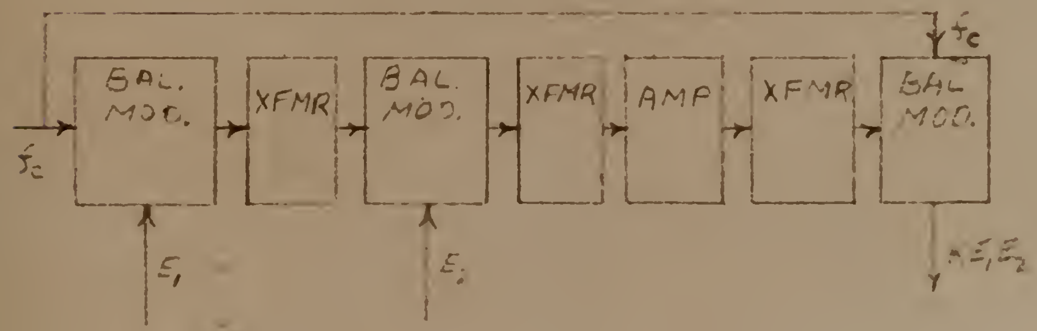
of which the term

$$K_6E_1E_2E_c^2\cos(w_c+w_1+w_2-w_c)t$$



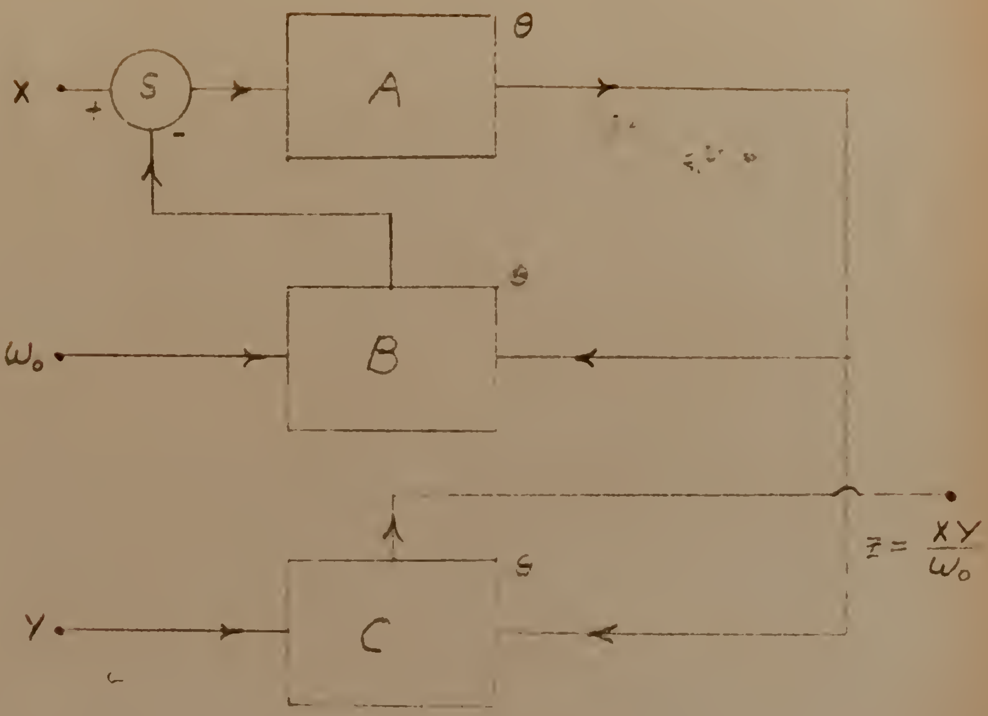

MIT FM-AM MULTIPLIER

FIGURE 8

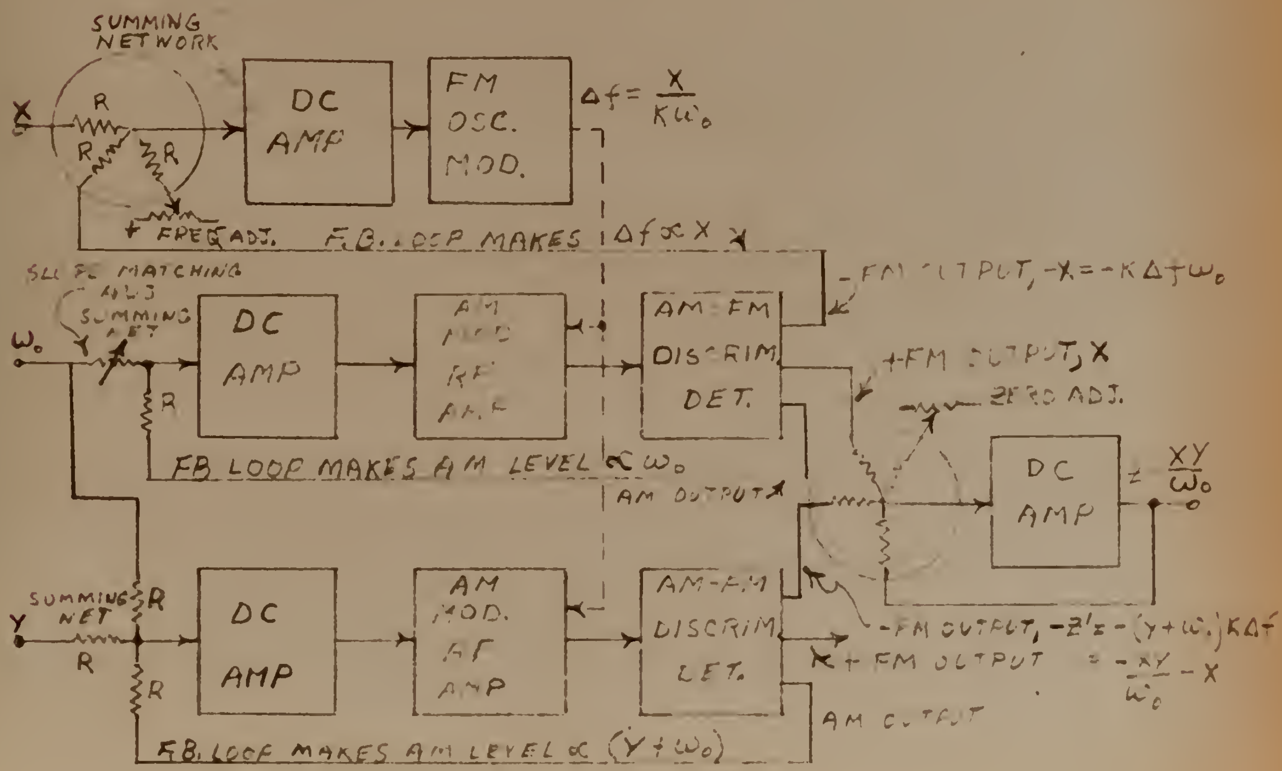



CAL-TECH AM-AM MULTIPLIER

FIGURE 9




ADJUSTER-ATTENUATOR MULTIPLIER
FIGURE 10




AM-FM MULTIPLIER BLOCK DIAGRAM
FIGURE 11

is the only one left after low pass filtering. Then

$E_{out} = K_7 E_1 E_2 \cos 2w_1 t$ if $E_c$ is held constant and $W_1 = W_2$.

Thus the output amplitude is proportional to the product $E_1 E_2$.

McCool (22) further develops an AM-FM double modulation multiplier which has definite possibilities for great accuracy. The basic theory of this device is illustrated by Figures 10 and 11.

Let $\theta$ = gain, adjusted such that

$$X - X' \doteq 0$$
$$X' = \theta W_o \doteq X$$
$$Z = \theta Y = \frac{XY}{W_o}$$

assuming all approximations hold.

X and Y are variables, $w_o$ is a fixed quantity

S = summing device

A = adjuster

B & C = attenuators

In this multiplier the two variables multiplied are the frequency modulation deviation and the carrier amplitude. $w_o$ is a fixed AC reference voltage. The carrier amplitude is made proportional to the modulation index of the AM wave, effectively making the device a four-quadrant multiplier. At present this multiplier has a long time computing error of plus or minus one per cent of full scale. According to McCool, future improvements are expected to reduce this error by a factor of 2 or 3. Unlike rectangular waveform multipliers, to be discussed briefly later, this multiplier requires no output filter as the only noise present is due to tube microphonics, power supply ripple, and drift.

All of these methods using modulation of a carrier require highly stable complicated circuits with many adjustments necessary for proper operation. They are capable of the highest degree of accuracy in the

analog multiplier field, and will be the analog answer for competing with digital circuits for multiplication. However, as stated before, if extreme accuracy is desired, the analog circuits become more complex, and the advantage of simplicity of design that analog multipliers have over digital devices is lost.

e. Rectangular area

There are almost as many devices utilizing the principle of rectangular area for multiplication as there are companies engaged in analog work. The most common type is best illustrated by the Boeing Multiplier, Figure 12. In this multiplier a phantastron generates a rectangular waveform with a repetition rate of 10KC controlled by crystal operated trigger source. The X input varies the duration of the positive going portion of the rectangular wave and the Y input varies the amplitude of the rectangular wave. The phase splitter converts the unbalanced time modulated waveform to a balanced waveform with respect to ground. This balance is for the purpose of giving a phase sensitive response to the amplitude modulator. Multiplication of these two variables is accomplished in the following manner (see Figure 13):

$t = t_o \quad K_1 X t_o$

$t_o = 50$ microseconds $= \tau/2$

(t varied between 20 and 80 microseconds by X input)

$t^+ = t_o + K_1 X t_o$ (duration of positive part of wave)

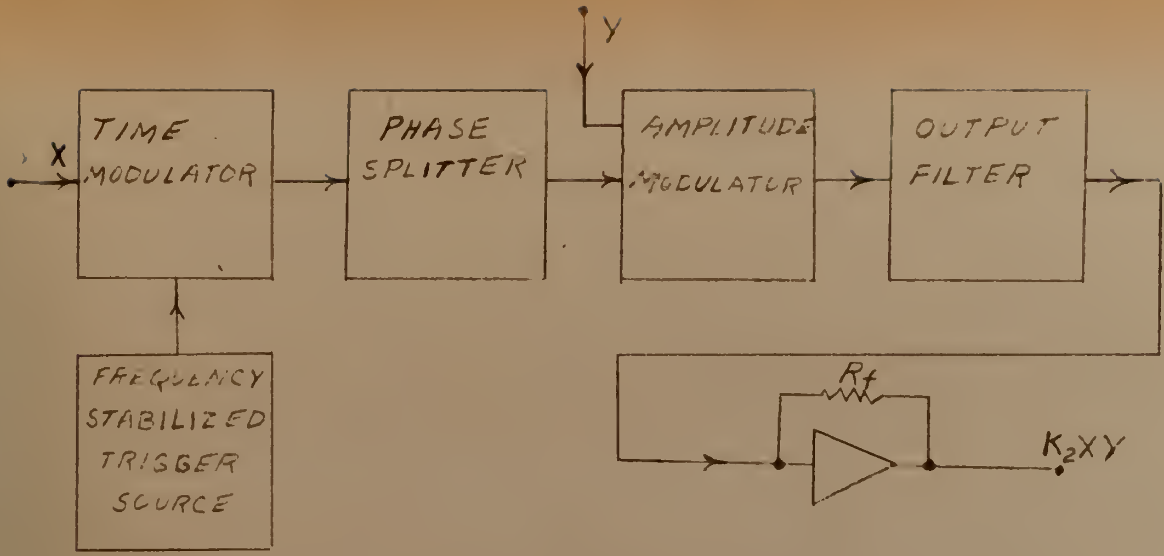
$t^- = t_o - K_1 X t_o$ (duration of negative part of wave)

$t^+$ amplitude $= +Y$

$t^-$ amplitude $= -Y$

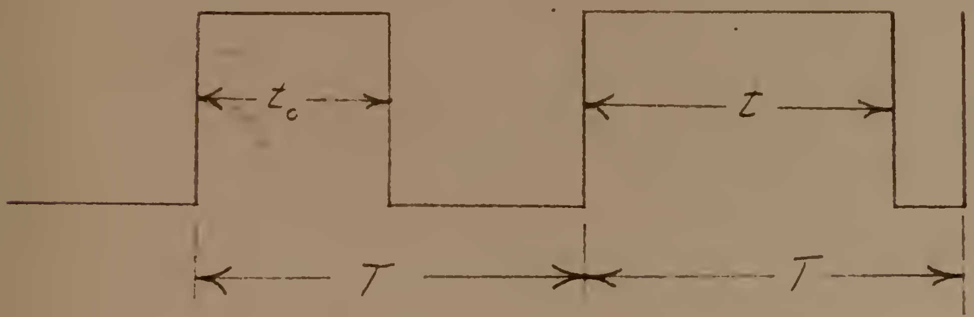
Area $^+ = Yt^+ = Y(t_o - K_1 X\ t_o)$

Area $^- = -Yt^- = -Y(t_o - K_1\ x\ t_o)$




BOEING MULTIPLIER

FIGURE 12




TIME MODULATED SQUARE WAVE

FIGURE 13

Net area for one cycle:

$$A = \bar{A} - \bar{A} = K_2XY$$

The output filter does the averaging by removing the carrier frequency component and all its harmonics.

This multiplier has an accuracy of about plus or minus two per cent of full scale and costs several hundred dollars. It requires two panel spaces in the computer rack, is a drain on the computer power supply, and although fairly stable it requires very careful adjustment for proper operation. It is a good competitive multiplier and is representative of this type of device.

f. Variable-gain amplifier

Very few multipliers have made use of the principle of variable gain so only a very brief discussion of this effect is given here. The usual method is for one variable to be applied to one control grid of a five grid tube and the other variable applied to the second control grid with the output of the tube roughly proportional to the product of the two inputs. As this device is very inaccurate it is seldom used. This is also only a two quadrant device. By this is meant that the input voltage may be either positive or negative with the output changing sign accordingly.

Tubes sometimes used in this application are the 6SA7 and 6AS6, both of them satisfying the relation

$$i_p = a e_{g1} e_{g3}$$

where $i_p$ is the plate current and $e_{g1}$ and $e_{g3}$ are the voltage inputs on grids one and three respectively. None of these devices is accurate over a very wide range and for all applications except for very approximate solutions they are not used.

g. Variable impedance

There are many ways of using the variable impedance principle in designing analog multipliers. Most of these applications are for electro-mechanical devices such as servo-driven potentiometers, and since these descriptions are of all-electronic multipliers the electro-mechanical and mechanical devices will not be discussed. A new all-electronic multiplier taking advantage of the principle of variable impedance, however, is described by Löfgren (17). He has developed a transistor multiplier utilizing the linear variation of collector resistance with varying emitter voltage. Using point contact transistors (Bell type 1698) and impressing a voltage on the emitter, it is found that collector resistance is constant over a fairly large range of collector current (or voltage) being dependent only on the voltage at the emitter. This resistance is a linear function of the emitter voltage within this range. (See Figure 14).

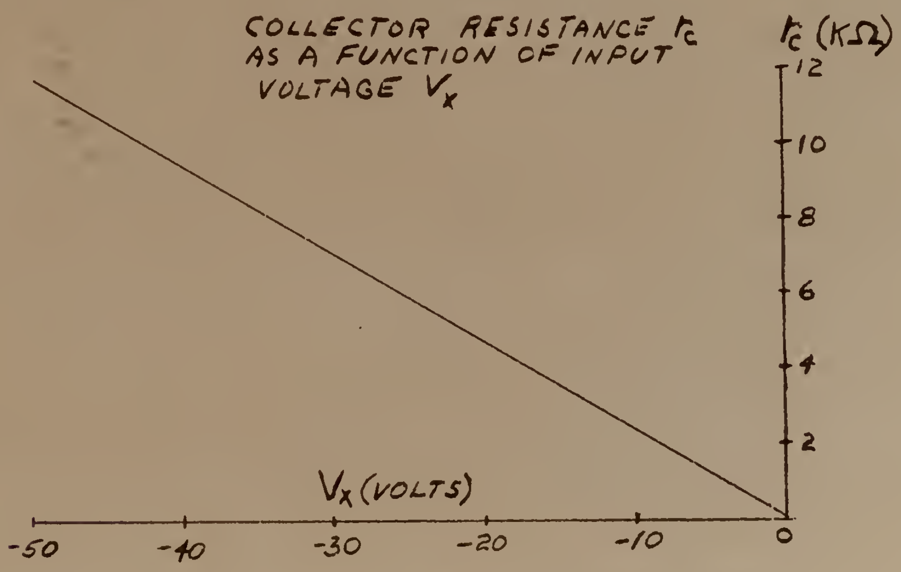
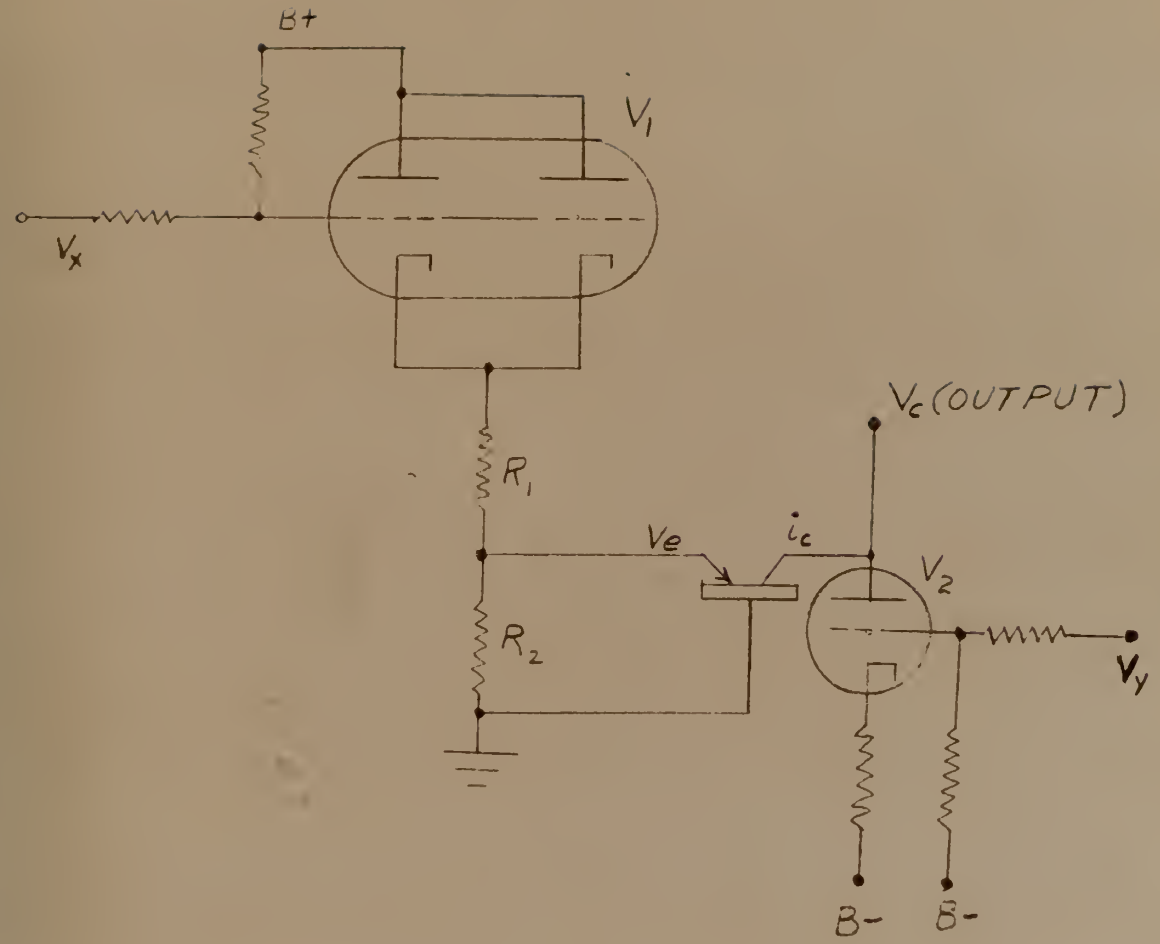



FIGURE 14

Using this property we can multiply. In Figure 15, $V_x$ and $V_y$ control $V_e$ and $i_c$ respectively; then $V_c$ will be proportional to the product $V_xV_y$. $V_y$ controls collector current via tube V2. Maximum error has been determined to be plus or minus 2 per cent in the value of $r_c$ by assuming it to be a linear function. Therefore, the maximum error in the multiplier is plus or minus 4 per cent since a plot of $V_c$ versus $V_y$ with $V_x$ as a parameter has an error deviation from straightness of not more than plus or minus 2 per cent. 4 volts seems to be the optimum range of variation of $V_c$ at present. Frequency response is good up to 100 kilocycles per second. $V_y$ can vary from zero to a positive value. $V_x$ cannot be zero. By adding constant voltages to the inputs to the output four quadrant operation is possible. This device is still experimental but it is the opinion of the author that it shows great promise.

3. Evaluation and Summary of Analog Multipliers

In the preceding pages the author has described many multipliers. These are only a few of the multitude of multipliers now in use or under development, but they serve to explain how the various principles of multiplication have been put to use, and to give a general background of analog multipliers before the Thyrite Multiplier is discussed. Two of the multipliers herein described, namely the Hall Effect Multiplier and the Transistor Multiplier, are experimental models, but the results of their tests show that they have definite possibilities for great usage in the field of analog computer work. It is of interest to compare the various types as to accuracy, frequency limitations, cost, size, and weight, as is done on page twenty-four.




TRANSISTOR MULTIPLIER

FIGURE 15

The next section reviews the work done by the author in the development of the Thyrite Multiplier. As can be seen on the chart on the next page, this multiplier is comparable in all respects, except that of extreme accuracy, to the other multipliers.

TYPICAL EXAMPLES

| Type | Accuracy % of f.s. | Frequency Limitations and Cause | Cost | Size and Weight |
|---|---|---|---|---|
| Photo-former | 1% | 150KC<br>CRT deflection circuits | Expensive | Large |
| Crossed Fields | 1% | 100KC<br>CRT deflection circuits | Expensive<br>(special tube required) | Large |
| Hall Effect | 1% | 20cps<br>Large inductance | No data | Large |
| Dual Modulation | 0.1% | 2-3KC<br>Depends on carrier frequency | Expensive | Large |
| Pulse Modulation | 0.5% | 2-300cps<br>Depends on carrier frequency | Fairly High | Fairly Large |
| Transistor | 4%<br>(% of value not f.s.) | 100KC | No data | Fairly Small & Light |
| Thyrite | 4% | 1KC<br>Sign changer amplifiers | Very Low | Very Small and Light |

SECTION II

A THYRITE MULTIPLIER

1. General Considerations

> It would be very desirable to have purely electronic multiplying devices of fair accuracy (0.1 to 0.5 per cent of full output scale, or 0.001 to 0.005 machine unit) and as simple in design as a good DC integrator (six or less vacuum tubes). (15)

This author feels that even more important now is the develpment of a fairly inexpensive multiplier of possibly plus or minus 2 to 4 per cent of full scale accuracy, and having ready adaptability to present computer racks and power supplies. Since the work carried out in this investigation was done at the Electronic Devices Group of the Industrial Products Division of the Boeing Airplane Company, it is not surprising that the design developed and described herein is such as to take full advantage of existing units of the Boeing Electronic Analog Computer.

The design considerations involved in the development of the Thyrite Multiplier were as follows:

a. Accuracy--plus or minus 2 per cent of full scale if possible.

b. Frequency response--as good as the rest of the computer units.*

c. Phase characteristics--as good as the rest of the computer units.

d. Size and weight--small.

e. Auxiliary equipment required--minimum.

f. Ease of operation and adjustment--simple.

g. Cost--low.

h. Power requirements--very low power drain.

i. Multiplying principle--quarter-square.

j. Squaring element--utilize the non-linear characteristics of Thyrite.

* The frequency response of the Boeing operational amplifiers is flat out to slightly beyond 1KC.

Since the specifications called for quarter-square operation, it was felt that the Boeing Four-Unit Sign Changer, a unit containing four direct-coupled amplifiers described later, could be used with the multiplier minimizing the added current drain on the power supply. Therefore some sort of addition and subtraction circuits should be designed using these amplifiers and obtaining the sum and difference of the two variables for squaring. The circuits finally arrived at are described in a later portion of this investigation. Briefly, these circuits are resistive networks with four diodes for four-quadrant operation. The squaring circuits had to use the Thyrite discs as their main element, and here again it was felt that the sign-changer amplifiers would be useful. In order to avoid switching difficulties, two identical (or nearly so) squaring amplifiers were decided upon. The circuits other than the amplifiers were also resistive networks. The problem at the outset then appeared to be:

Given: Four DC amplifiers, design circuits such that four-quadrant multiplication can be achieved using the method of quarter-squares.

It should be possible that the external circuits could be mounted on a terminal board and plugged into the Four-Unit Sign Changer, thus making a small compact unit.

The purpose of this design is not to replace but merely to add to the list of Boeing computer products a multiplier of fair accuracy and low cost to fulfill the need for approximate solutions of differential equations and observe the results without excessive calibration procedures.

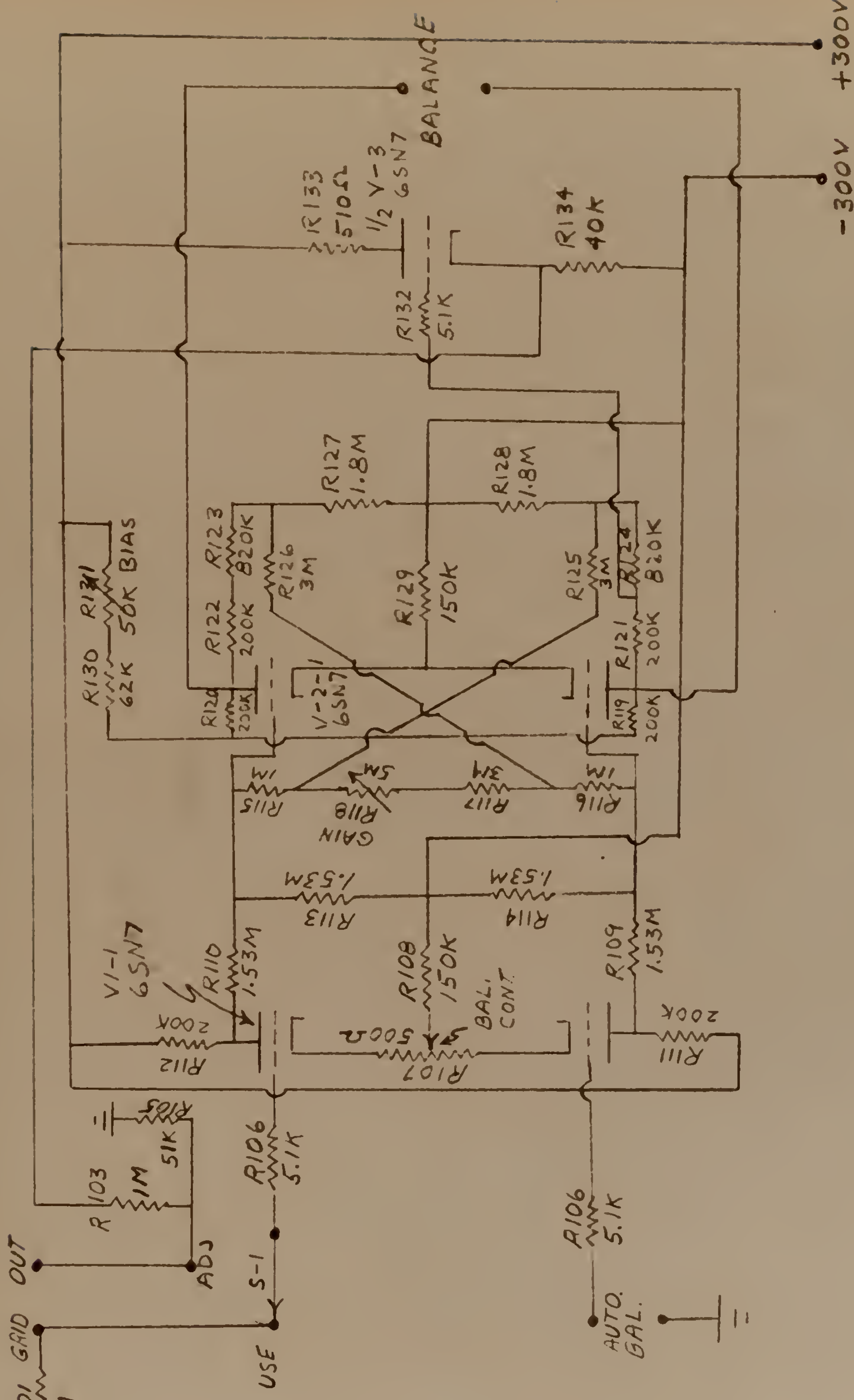
2. Four-unit sign changer

The Boeing Electronic Analog Computer has as a part of its installed

equipment a device called the "Four-unit sign changer". This unit contains four direct-coupled amplifiers, each one similar in electronic circuitry to the standard Boeing Operational Amplifier. All four amplifiers are housed in one chassis. The front panel of the sign changer contains jacks connected to the grids and outputs of the amplifiers. It was decided to use this unit with its four amplifiers as a part of the Thyrite Multiplier using quarter-square method of multiplication. The passive circuit elements were mounted on a sheet of phenolic material with banana plugs connecting the components on this sheet directly to the banana jacks on the sign changer. A complete circuit diagram of the sign changer appears in Figure 16.

Each amplifier consists of a phase inverter input stage, a balanced high gain stage, and a cathode follower output stage. Tube V-1 is the phase inverter and the potentiometer R107 is adjusted for a balanced output. The outputs of V-1 are directly coupled to the grids of V-2 by the resistor network. R118 adjusts the gain of V-2 by controlling the magnitude of the positive feedback to the grids. The output of V-2 is directly coupled to the grid of V-3 which acts to prevent loading of the amplifier by a low impedance shunted across the output. The bias adjustment, R131 sets the proper no signal bias level of V-2. The bias and gain adjustments need not be made more frequently than once a month. The balance control which compensates for amplifier drift is not required to be adjusted in this multiplier application as the compensating auto-balance unit described below takes over its function on a continuous basis.

As in all high gain direct-coupled amplifiers there is a tendency for the sign changer amplifiers to drift slowly. Drift may be defined as a varying output with no signal input and the input resistor grounded.



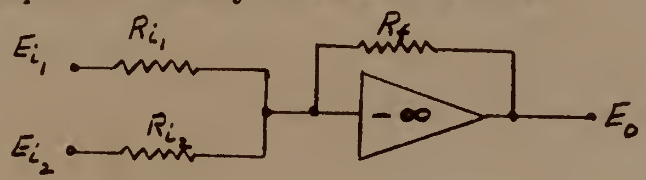

BOEING FOUR-UNIT SIGN CHANGER
ONE OF FOUR AMPLIFIER CIRCUITS
(ALL FOUR IDENTICAL)

FIGURE 16

With perfect balance the output under these conditions should be zero, but the high open loop gain of these amplifiers causes an output due to any slight unbalance. Therefore automatic balancing is required. The Boeing sequencing auto-balance unit was used for this purpose and kept the drift output to a value less than one tenth of a volt. The sequencing auto-balance unit samples at a rapid rate sixteen separate amplifier channels consecutively by means of a commutating motor driven switch. As each amplifier channel is sampled, a compensating amplifier in the unit brings the sampled amplifier back to a condition of zero balance (plus or minus 0.1 volt).

3. Summing circuits

Summing amplifier theory in effect is this:




Assumptions:

$i_g$ (grid current) $= 0$

$|A| >> 1$ (open loop)

$|A| >> R_f\left(\frac{1}{R_{i_1}} + \frac{1}{R_{i_2}}\right)$

For node A:

a. $$(E_g - E_o)\frac{1}{R_f} + (E_g - E_{i_1})\frac{1}{R_{i_1}} + (E_g - E_{i_2})\frac{1}{R_{i_2}} = 0$$

b. $$E_g = \frac{E_o}{A}$$

c. $$\left(\frac{E_o}{A} - E_o\right)\frac{1}{R_f} + \left(\frac{E_o}{A} - E_{i_1}\right)\frac{1}{R_{i_1}} + \left(\frac{E_o}{A} - E_{i_2}\right)\frac{1}{R_{i_2}} = 0$$

d. $$E_o = \left(\frac{E_{i_1}}{R_{i_1}} + \frac{E_{i_2}}{R_{i_2}}\right)\left(\frac{AR_f}{1 - A + R_f\left(\frac{1}{R_{i_1}} + \frac{1}{R_{i_2}}\right)}\right)$$

$$e_o \qquad E_o \doteq \left( \frac{E_{i_1}}{R_{i_1}} + \frac{E_{i_2}}{R_{i_2}} \right) R_f$$

Now, using equation e. and referring to Figure 17, we can see how the sum and difference of the input variables is obtained for use in the quarter-square Thyrite Multiplier. Points "a" are the summing points for the inputs A and B. Since all $R_1$'s are equal valued resistors and all $R_2$'s are equal valued resistors, the summing points "a" represent the sum (A+B). The amplifier shown has a gain of one half

$$\tfrac{1}{2}R_2 \div R_2 = \tfrac{1}{2}$$

(feedback resistor is one half the series grid resistance) so point "b" is at a potential of $-\frac{(A+B)}{2}$

Point "c" is the summing point of A and $-\frac{(A+B)}{2}$

and represents $\frac{2A-A-B}{2} = \frac{A-B}{2}$

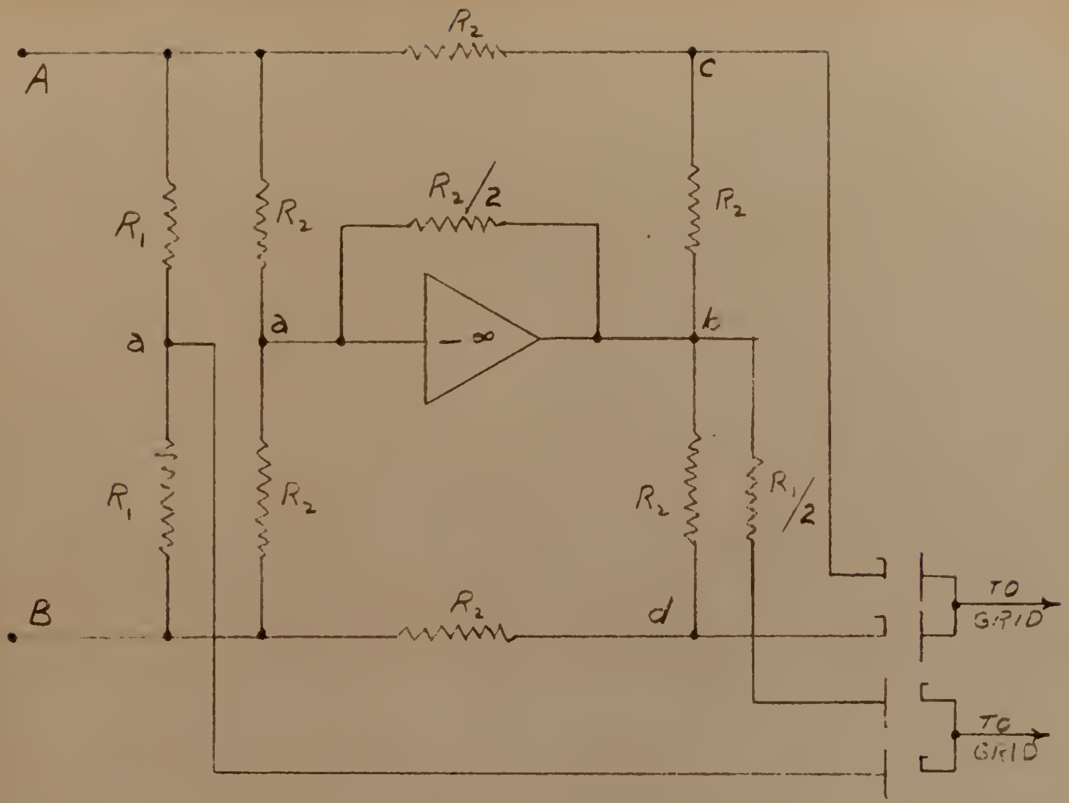
Point "d" is the summing point for B and $-\frac{(A+B)}{2}$

and represents $\frac{2B-A-B}{2} = \frac{B-A}{2}$

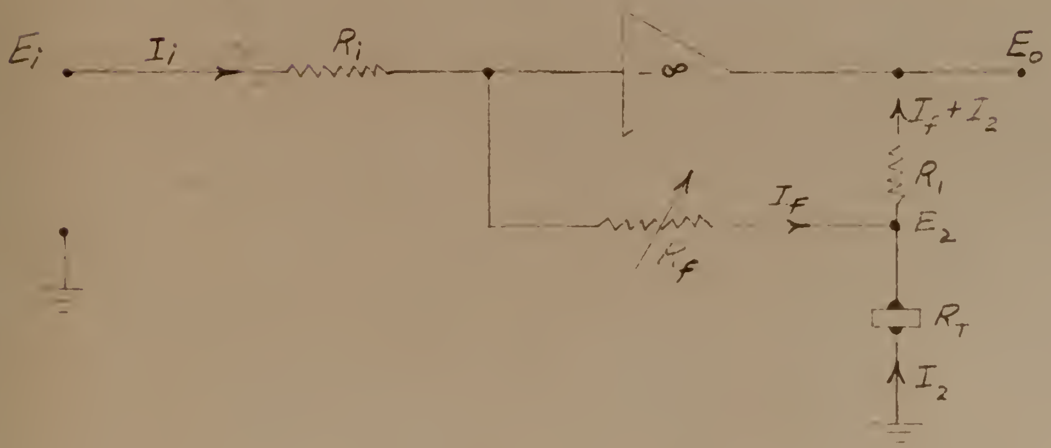
Note that although the negative sum is half the positive sum, the input resistor $\frac{R_1}{2}$ effectively doubles this quantity in the following amplifier, thus making the output the same for both positive and negative sums. Thus the diodes are summing points for the following:

$$(A+B),\ -(A+B),\ \frac{(A-B)}{2},\ \frac{(B-A)}{2}$$

A good multiplier should be capable of four quadrant operation. That is, the algebraic sign of the output product should be correct for the particular algebraic signs of the inputs. Inputs of the same algebraic sign should have a positive product; inputs of opposite sign should have a negative product. At the diodes sums and differences of both algebraic signs are available and the diode halves which conduct cause the output to be of proper sign. Let us examine this further.




SUMMING CIRCUITS
FIGURE 17




SQUARING CIRCUIT
FIGURE 18

Suppose both A and B are positive, the plate of the diode connected to point "a" will be positive to the cathode which is connected to a grid of zero potential (always). This tube will conduct bringing the plate to zero potential, or nearly so, and "a" will be the summing point. Point "b" will be negative (because of change of sign through amplifier) and the diode connected to this point will not conduct.

If A and B are of opposite sign (B negative) with $|B|$ greater than $|A|$, point "a" will be negative and the diode connected to "b" will conduct. Thus the input to the amplifier connected to the sum circuit is always positive. Similarly, the input to the difference circuit is always negative because of the reversal of connections for the difference diodes. The following two amplifier circuits (Figure 21) through which the variables pass, cause two reversals of sign such that their outputs after being squared in magnitude and summed are

$$\frac{(A+B)^2 - (A-B)^2}{8} = \frac{AB}{2}$$

Squaring is accomplished in a circuit to be described later.

Now let us analyze the output for proper algebraic sign. We have seen that the sum (A+B) is always positive and the difference (A-B) is always negative. Therefore when adding the sum and the difference it must be the magnitude of each that determines the algebraic sign of the output.

$$(A+B)^2 - (A-B)^2 = 4AB$$

Case I: $A > B$, $A+$, $B+$

$$(A\overset{+}{+}B)^2 > (A\overset{-}{-}B)^2 = +$$

Case II: $A < B$, $A+$, $B+$

$$(A\overset{+}{+}B)^2 > (A\overset{-}{-}B)^2 = +$$

Case III: $|A| > |B|$, $A+$, $B-$

$$(A\overset{+}{+}B)^2 < (A\overset{-}{-}B)^2 = -$$

Case IV: $|A|<|B|$, $A+$, $B-$

$$(A \overset{+}{+} B)^2 < (A-B)^2 = -$$

Thus the output has the proper sign.

At first it was thought that selenium rectifiers or germanium diodes could be used to ensure four-quadrant operation since no filament leads would be required. However, it was found after measurement and consultation of published data on these components that their back resistance was far too low for this multiplier. What was needed was almost an infinite back resistance (i.e. open circuit) since any back current would cause large errors in the multiplying process.

4. Squaring circuits

The heart of the quarter-square multiplier is the squaring circuit. The requirements for the squaring circuits in the Thyrite Multiplier were to take voltages from zero to fifty volts and transform them in such a way that the outputs of the squaring amplifiers were proportional to the square of the inputs. Proper scaling kept the outputs in the linear range of the amplifiers (0 to plus or minus 50 volts). For this multiplier the squaring amplifiers depend on the non-linear characteristics of Thyrite to generate the square law action.

Thyrite (2, 3, 4, 11, 13, 23), a General Electric trade mark, is a mixture of silicon carbide in a ceramic binder formed at high pressure into a solid shape. After the high pressure operation a firing procedure at high temperatures (approximately 1200 $^{0}$C) takes place. Leads make contact with this material through a metal coating that is sprayed on the outside surfaces. Thyrite is bilateral; that is, it has the same effect on a current passing through it in either direction as it is only a resistance material. Thyrite has a low thermal conductivity, and its

resistance is reduced at temperatures higher than normal at a rate of about one per cent per degree C rise in temperature for a constant voltage across it. However, in this application the effect of temperature change can be corrected by the feedback potentiometer the action of which will be explained. An equation expressing the action of Thyrite is:

$I = KE^n$ where

$I$ = current through the sample

$E$ = voltage across the sample

$K$ and $n$ are constants for a particular piece of Thyrite.

The E - I characteristics approximate a straight line on log-log graph paper over a considerable range of current, but below 200 to 300 volts per inch of thickness the n value reduces until at very low voltages the E - I curve approaches the 45 degree linear resistance slope. K is the current at one volt and n is the slope of the E - I characteristic. That is, n is the cotangent of the angle formed by the E - I curve and the I axis.

The resistance of Thyrite is always positive and depends only on the magnitude of the voltage across the sample. Although there are several methods listed in the references for measuring the Thyrite constants, this author took the published characteristic curves of Thyrite as a starting point in the development of a squaring amplifier. It was found that the one-half inch discs of Thyrite, G. E. Catalog Number 8396839G1, best filled the requirements for this circuit. The K was $7.2 \times 10^{-5}$ and n was 2.2. Figure 19 shows these characteristic curves and figure 20 shows the range of variation of actual Thyrite discs of this type. Notice that the general shape of the E - I curve is main-

tained with a shift only in ordinate.

The actual squaring circuit is shown in Figure 18. In this circuit the following assumptions are made:

$$i_g = 0$$

$$E_g = \frac{E_o}{A} \doteq 0$$

Then:

a. $\dfrac{E_i}{R_i} = \dfrac{E_2}{R_f}$

b. $I_2 = KE_2^n$

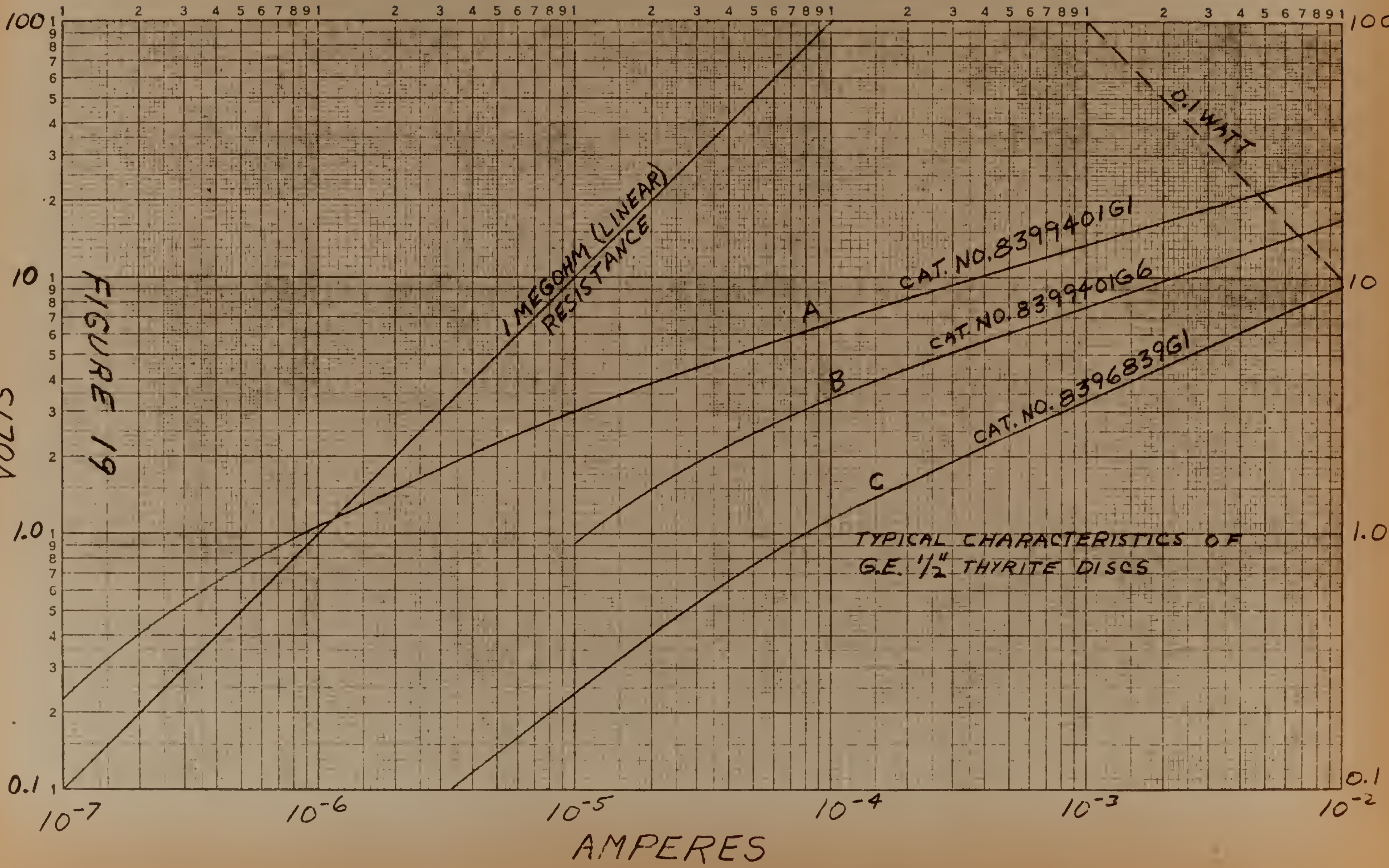
c. $I_f = \dfrac{E_2}{R_f}$

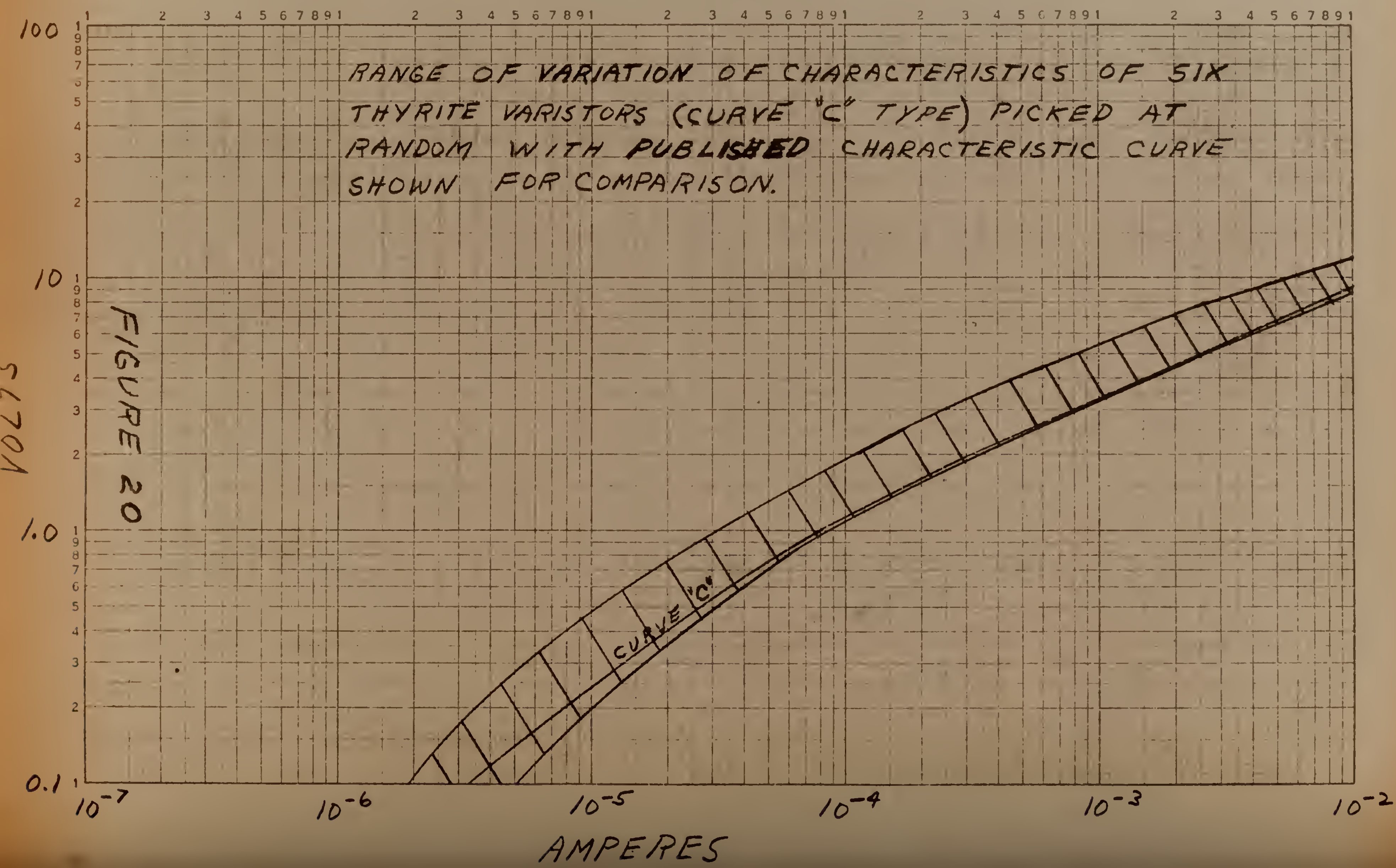
d. $E_o = E_2 + (I_2 + I_f)R_1$

e. $E_o = E_2 + \left(KE_2^n + \dfrac{E_2}{R_f}\right) R_1$

f. $E_o = \dfrac{R_f}{R_i} E_i + R_1 \quad \dfrac{E_i}{R_i} \quad + K\left(\dfrac{R_f}{R_i} E_i\right)^n$

Now at first glance this does not appear to give a square law output. However, in Appendix A is shown the method by which the output of this circuit was compared to the output of a "perfect" squaring device with good results. Once the circuit constants were determined in this manner, these values were substituted into the theoretical expression for the squaring circuit and the answers closely approached the measured values. Differences were mainly due to the fact that both K and n, the Thyrite constants, change throughout the range of input voltage and are constant for only a small portion of this range. The maximum error observed was eight-tenths of a volt, and the maximum computed error


0.1 WATT
1 MEGOHM (LINEAR) RESISTANCE
CAT. NO. 8399401G1
CAT. NO. 8399401G6
CAT. NO. 8396839G1
A
B
C
TYPICAL CHARACTERISTICS OF
G.E. 1/2" THYRITE DISCS
VOLTS
AMPERES
100
10
1.0
0.1
$10^{-7}$
$10^{-6}$
$10^{-5}$
$10^{-4}$
$10^{-3}$
$10^{-2}$
FIGURE 19


RANGE OF VARIATION OF CHARACTERISTICS OF SIX THYRITE VARISTORS (CURVE "C" TYPE) PICKED AT RANDOM WITH PUBLISHED CHARACTERISTIC CURVE SHOWN FOR COMPARISON.
FIGURE 20
CURVE "C"
VOLTS
AMPERES
100
10
1.0
0.1
$10^{-7}$
$10^{-6}$
$10^{-5}$
$10^{-4}$
$10^{-3}$
$10^{-2}$

was also eight-tenths of a volt, although the input was varied from zero to fifty volts. Appendix B shows a tabulation of the errors determined. Thus, although the theoretical expression for the squaring circuit does not show perfect square-law action, the circuit does produce an output curve with maximum errors of about plus or minus 4 per cent of full scale which is within the tolerance of the modified specifications.

5. Complete Multiplier

Now that a squaring circuit had been found it was possible to construct an actual multiplier. At this time an explanation of scaling will be in order. The design should be such that the full scale capabilities of the amplifiers are utilized to thereby minimize drift. Therefore to represent numbers from zero to ten, the inputs ranged from zero to fifty volts. That is 5A and 5B are the inputs. It was found that a twenty five volt maximum output from the squaring amplifiers gave the least error so for inputs to the sum channel of $5(A+B)$ the output was $\frac{(A+B)^2}{16}$, and for the difference channel $5(A - B)$ gave an output of $\frac{(A-B)^2}{16}$. The final summing amplifier shown in the complete circuit diagram in Figure 21, had a gain of two to make full use of the 50 volt maximum output. The final output is

$$\frac{(A+B)^2}{8} - \frac{(A-B)^2}{8} = \frac{AB}{2}$$

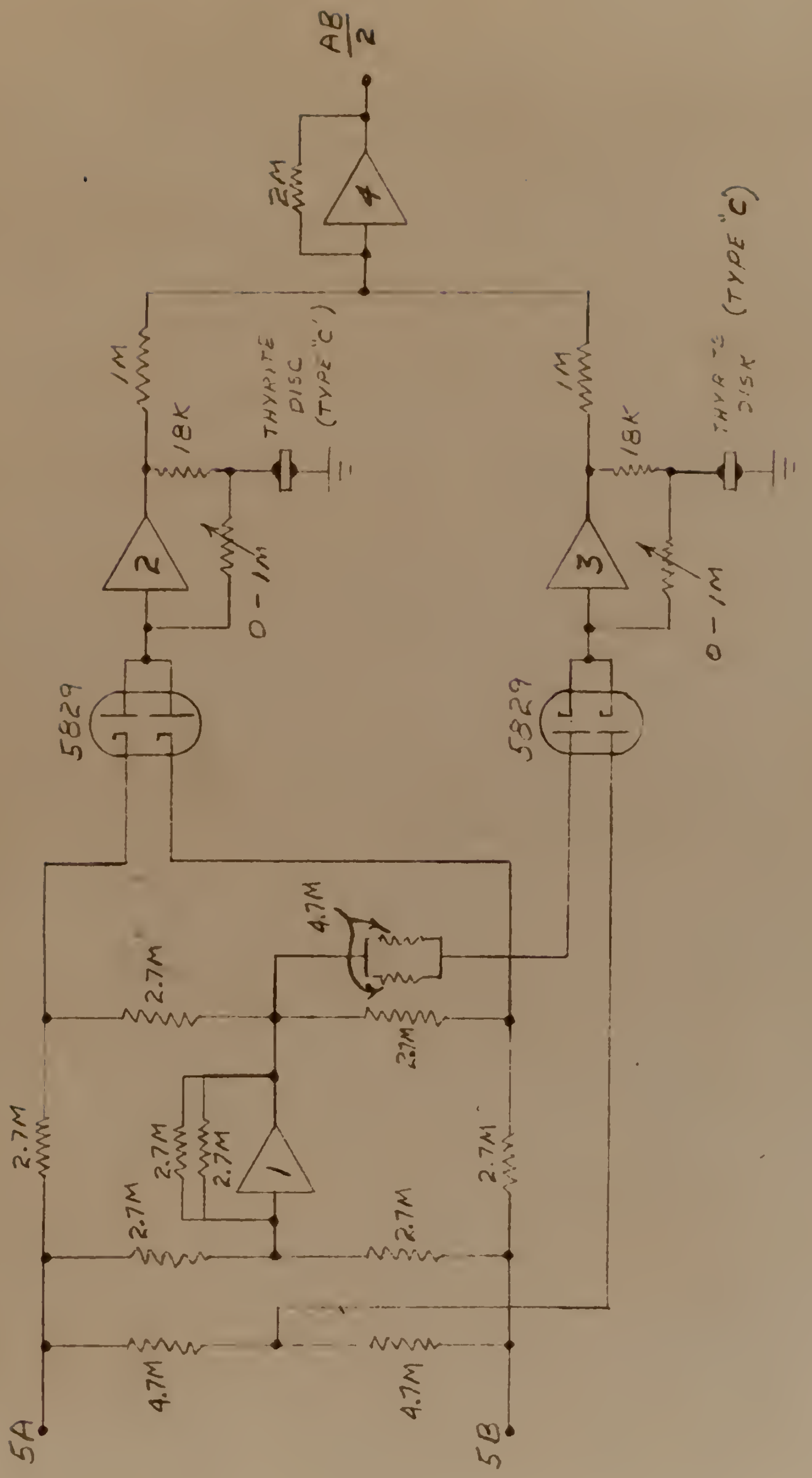
For $A = 10$ and $B = 10$ the output is 50 volts and the A and B channel inputs are also 50 volts.

With the multiplier designed and constructed, tests could be run on the complete system. The first tests were DC tests---multiplication of two DC voltages. All integral values of A and B from zero to ten

were tried and the output plotted in relation to the inputs. This is shown in Figure 22, with the true products drawn for comparison. A change in the algebraic sign of either input made very little difference in the output except for sign since the magnitude of the inputs to the squaring amplifiers was essentially unchanged. Figure 23 shows this same information in a different form. In these curves the actual voltage error is shown for various combinations of inputs. Remember that the actual inputs are 5A and 5B.

The next tests to be conducted were frequency response and phase lag of the multiplier. For the frequency response the same 10 volt RMS sine wave was impressed on both A and B channels by means of an audio oscillator and isolating amplifier. The output was measured on a vacuum tube voltmeter and plotted in Figure 24, where the ratio of output to input in db variation from the lowest frequency is shown versus frequency. The sharp rise after one kilocycle per second is due to the characteristics of the sign changer amplifiers whose frequency and phase characteristics are plotted in Figures 25 and 27. The multiplier itself had no noticeable frequency limitations within the range of the amplifiers.

The phase characteristics of the multiplier were measured with a DC input of 46 volts to one channel and a 10 volt RMS sine wave input to the other channel. The difference in phase between the output and the sine wave input was measured with an electronic phase meter and plotted versus frequency in Figure 26. Compared to most multipliers with frequency limitations of only a few hundred cycles this Thyrite Multiplier response is quite good. Since the operational amplifiers of the Boeing Analog Computer have the same frequency limitations as the multiplier, the Thyrite Multiplier is as good in this respect as the complete system.




THYRITE MULTIPLIER

FIGURE 21

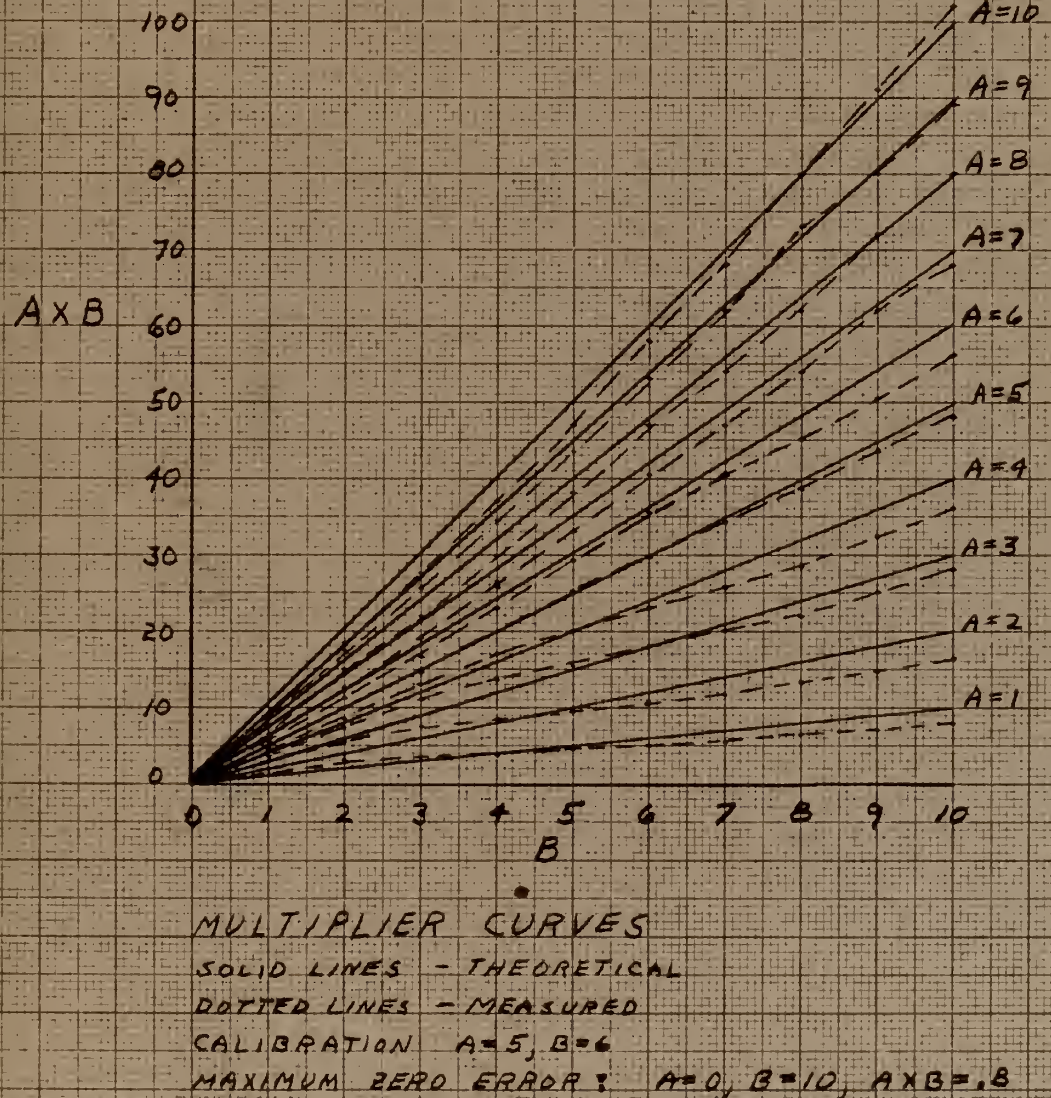

A=10
A=9
A=8
A=7
A=6
A=5
A=4
A=3
A=2
A=1
A x B
100
90
80
70
60
50
40
30
20
10
0
0 1 2 3 4 5 6 7 8 9 10
B
MULTIPLIER CURVES
SOLID LINES - THEORETICAL
DOTTED LINES - MEASURED
CALIBRATION A=5, B=6
MAXIMUM ZERO ERROR: A=0, B=10, AXB=.8

FIGURE 22

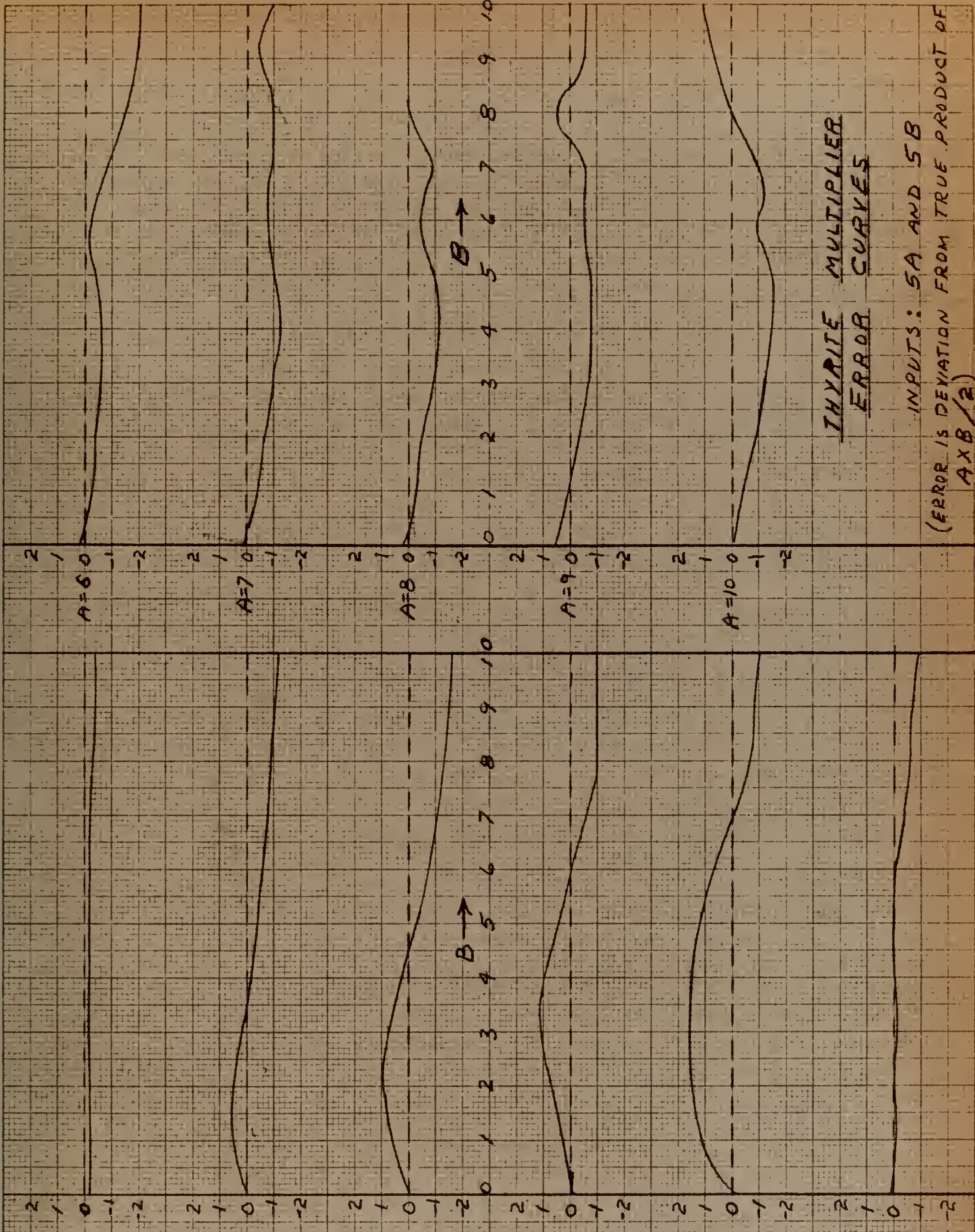

THYRITE MULTIPLIER
ERROR CURVES
INPUTS: 5A AND 5B
(ERROR IS DEVIATION FROM TRUE PRODUCT OF A×B/2)
ERROR (VOLTS)
B →
A=0
A=1
A=2
A=3
A=4
A=5
A=6
A=7
A=8
A=9
A=10

FIGURE 23

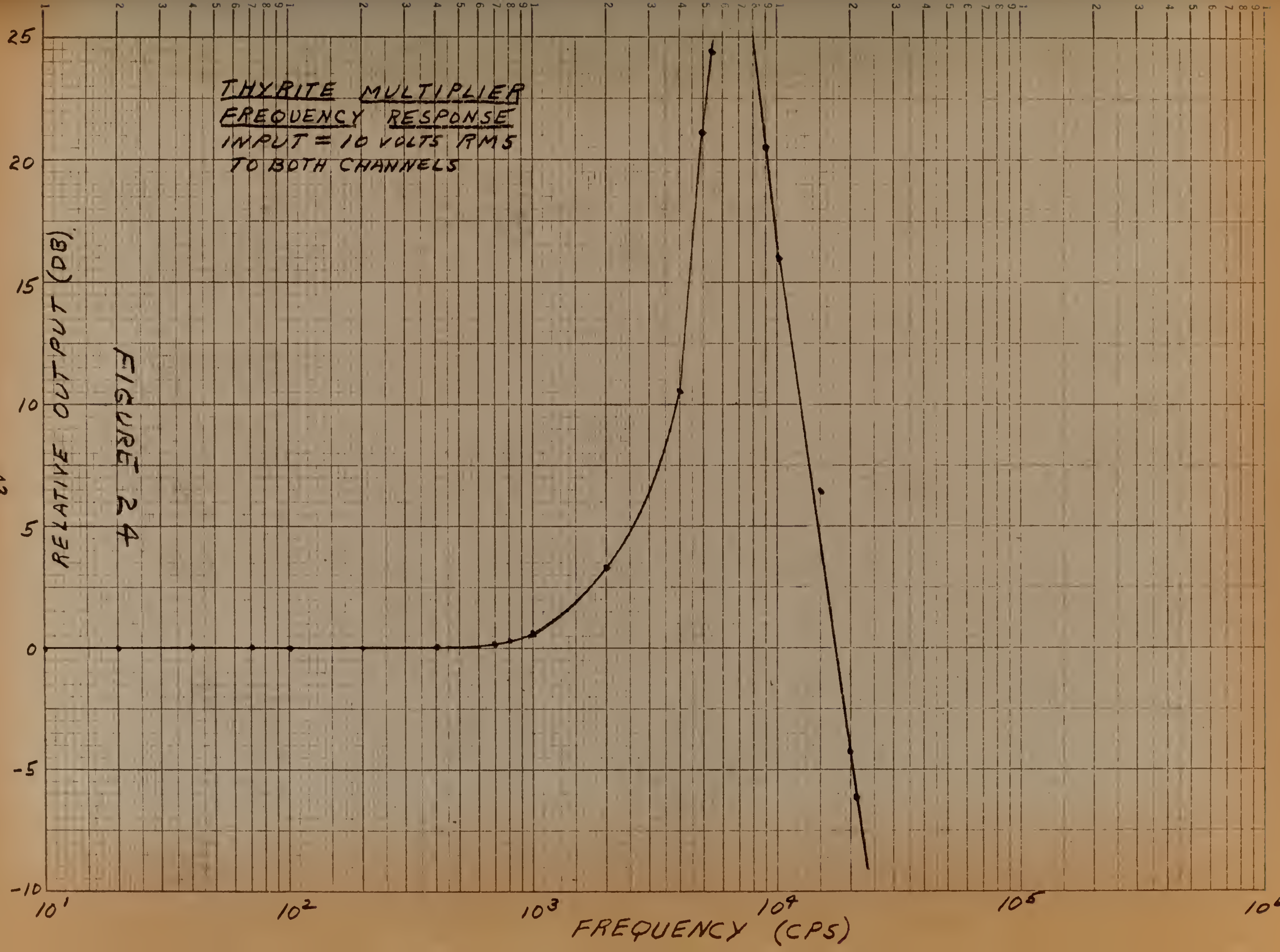
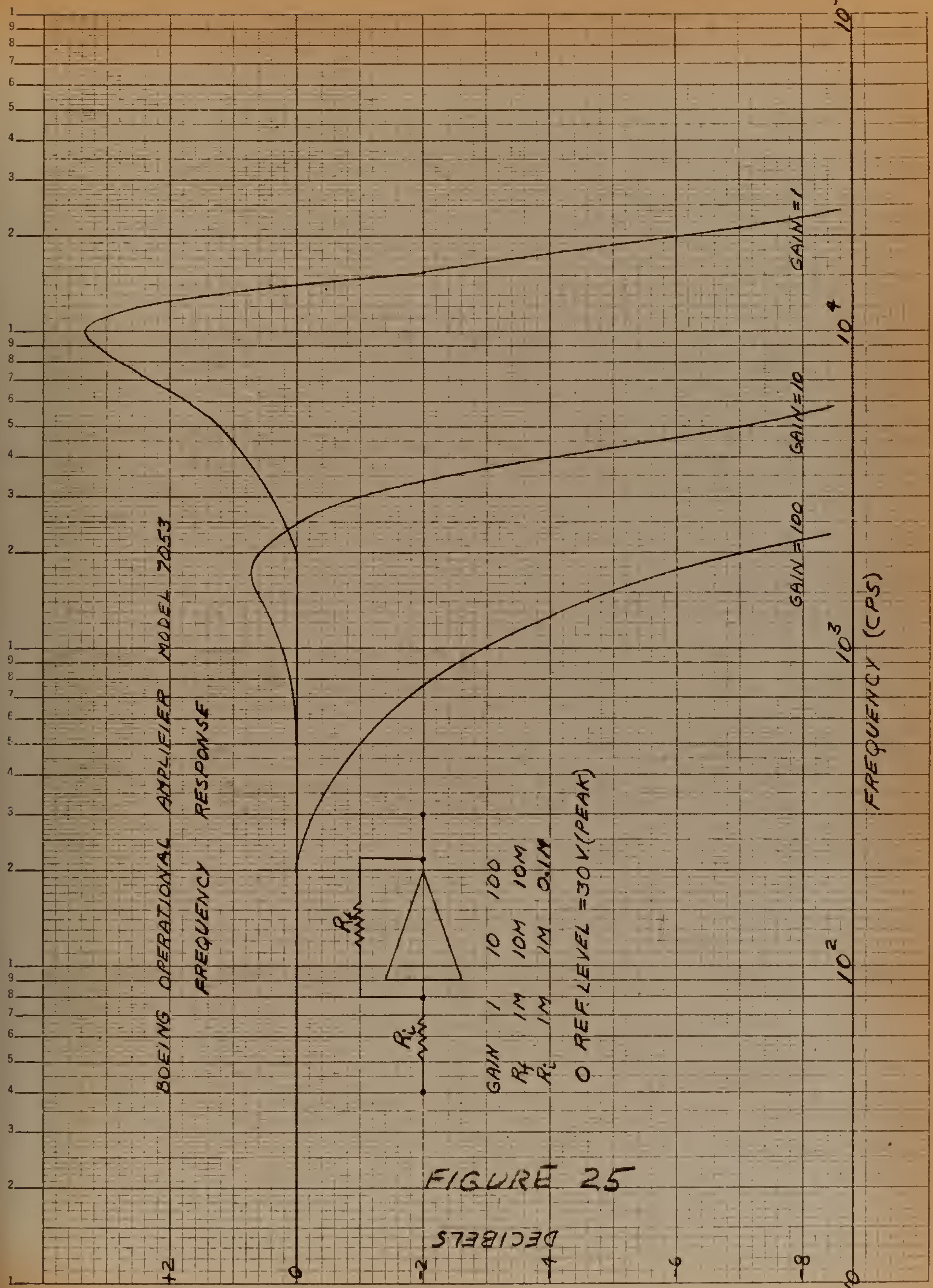

THYRITE MULTIPLIER
FREQUENCY RESPONSE
INPUT = 10 VOLTS RMS
TO BOTH CHANNELS
RELATIVE OUTPUT (DB)
25
20
15
10
5
0
-5
-10
$10^1$
$10^2$
$10^3$
$10^4$
$10^5$
$10^6$
FREQUENCY (CPS)

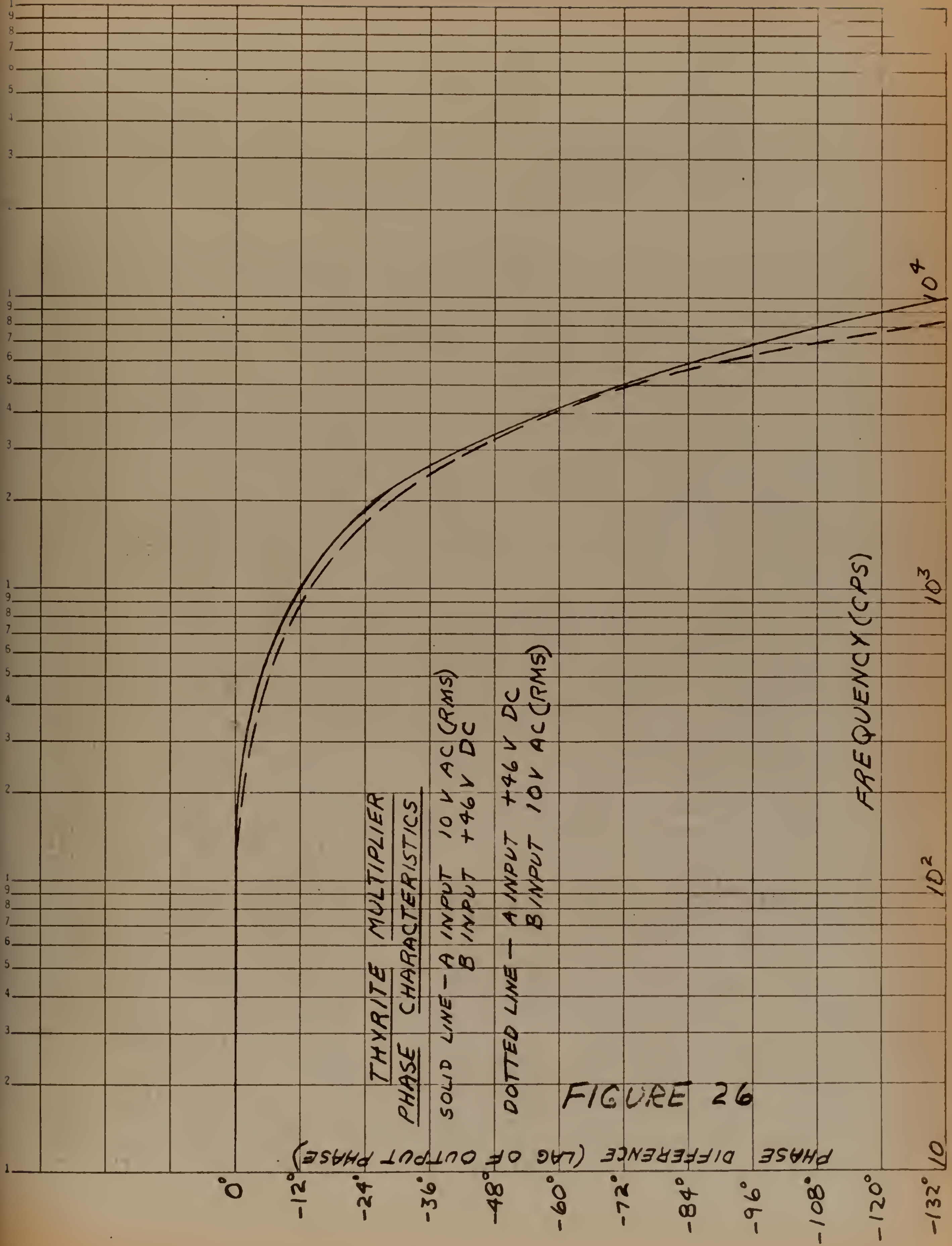
FIGURE 24

The next test was to see if a sine wave impressed on both channels would give the proper product. That is:

$$\sin wt \times \sin wt = \sin^2 wt = \tfrac{1}{2}(1-\cos 2wt)$$

Thus the output should have a DC component and a sinusoidal component of double frequency. This output was observed on a cathode ray oscilloscope and was composed of the expected DC component and the double frequency component, although the sinusoid was slightly distorted due to errors in the multiplier squaring circuits.


BOEING OPERATIONAL AMPLIFIER MODEL 7053
FREQUENCY RESPONSE
$R_f$
$R_i$
GAIN 1 10 100
$R_f$ 1M 10M 10M
$R_i$ 1M 1M 0.1M
0 REF. LEVEL = 30 V. (PEAK)
GAIN = 1
GAIN = 10
GAIN = 100
FIGURE 25
DECIBELS
+2
0
-2
-4
-6
-8
10
$10^2$
$10^3$
$10^4$
$10^5$
FREQUENCY (CPS)


THYRITE MULTIPLIER
PHASE CHARACTERISTICS
SOLID LINE — A INPUT 10 V AC (RMS)
B INPUT +46 V DC
DOTTED LINE — A INPUT +46 V DC
B INPUT 10 V AC (RMS)
FREQUENCY (CPS)
PHASE DIFFERENCE (LAG OF OUTPUT PHASE)
0°
-12°
-24°
-36°
-48°
-60°
-72°
-84°
-96°
-108°
-120°
-132°
10
$10^2$
$10^3$
$10^4$
FIGURE 26

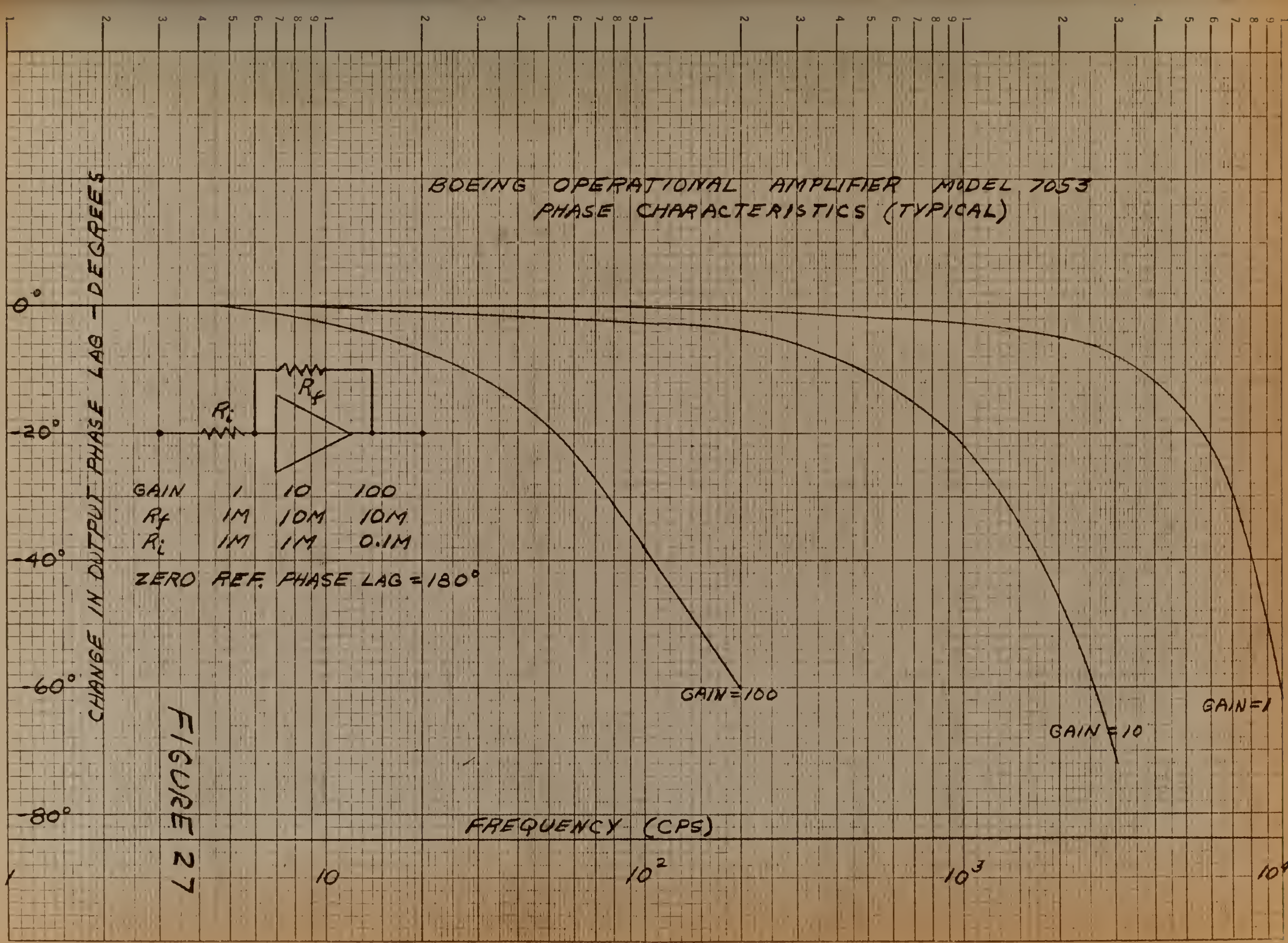

BOEING OPERATIONAL AMPLIFIER MODEL 7053
PHASE CHARACTERISTICS (TYPICAL)
CHANGE IN OUTPUT PHASE LAG - DEGREES
0°
-20°
-40°
-60°
-80°
$R_i$
$R_f$
GAIN 1 10 100
$R_f$ 1M 10M 10M
$R_i$ 1M 1M 0.1M
ZERO REF. PHASE LAG = 180°
GAIN=100
GAIN=10
GAIN=1
FREQUENCY (CPS)
1
10
$10^2$
$10^3$
$10^4$

FIGURE 27

SECTION III

CONCLUSIONS

The completed version of this Thyrite Multiplier has a maximum error of plus or minus 4 per cent of full scale or two volts, and a flat frequency response out to 1000cps. The unit is small, of low cost, and uses an existing computer component--the four unit sign changer. The only added load on the power supply is the filament current for the four-quadrant sub-miniature diodes. The only external connections required (except for input and output leads) are the filament and ground leads which connect by cable to the computer power bus. Thus the unit is quickly attached and put into operation. With auto-balance in use, the only adjustments required are the two feedback potentiometers which calibrate with DC input voltages to a zero error output in the mid-range of the multiplier. Accuracy of the multiplier is limited only by the Thyrite discs themselves as all resistors in the summing circuit and the output amplifier are of the precision type. These resistors are Welwyn deposited carbon precision resistors since they were found to be within plus or minus 0.1 per cent of their desired values although they were rated at plus or minus 1 per cent.

The multiplier is so arranged that servicing is very simple. All parts are mounted on a phenolic board which will have a metal cover. Removal of the cover exposes all components for voltage measurements or replacement.

The advantages of this multiplier as compared to other multipliers now in production may be briefly summarized as follows:

Accuracy ---------------Fair (some multipliers have up to 10% error)

Frequency response-------Excellent

Phase shift---------------------Excellent

Size----------------------------Excellent

Weight--------------------------Excellent

Cost----------------------------Excellent

Ease of use---------------------Excellent

Simplicity of servicing---------Excellent

There are no multipliers comparable to this Thyrite Multiplier in size, weight, cost, ease of use, and simplicity of servicing. Therefore it is the opinion of the author that this multiplier should have wide usage in the field of analog computers.

# APPENDIX A

## SQUARING ERROR TESTS

In the design and development of the squaring circuit some way was needed to rapidly and accurately test for proper squaring operation. Figure 28 shows the actual test setup. The ganged precision potentiometers have an accuracy within plus or minus 0.1 per cent, and no measurable error in this circuit was found.

For an input to the first amplifier of 2.5X the output of the second potentiometer is

$$\frac{2.5X}{50}(2.5X)$$

or

$$\frac{6.25X^2}{50}$$

This output is halved in the second amplifier by reason of the gain of 1/2, $(R_f/R_1 = 1/2)$

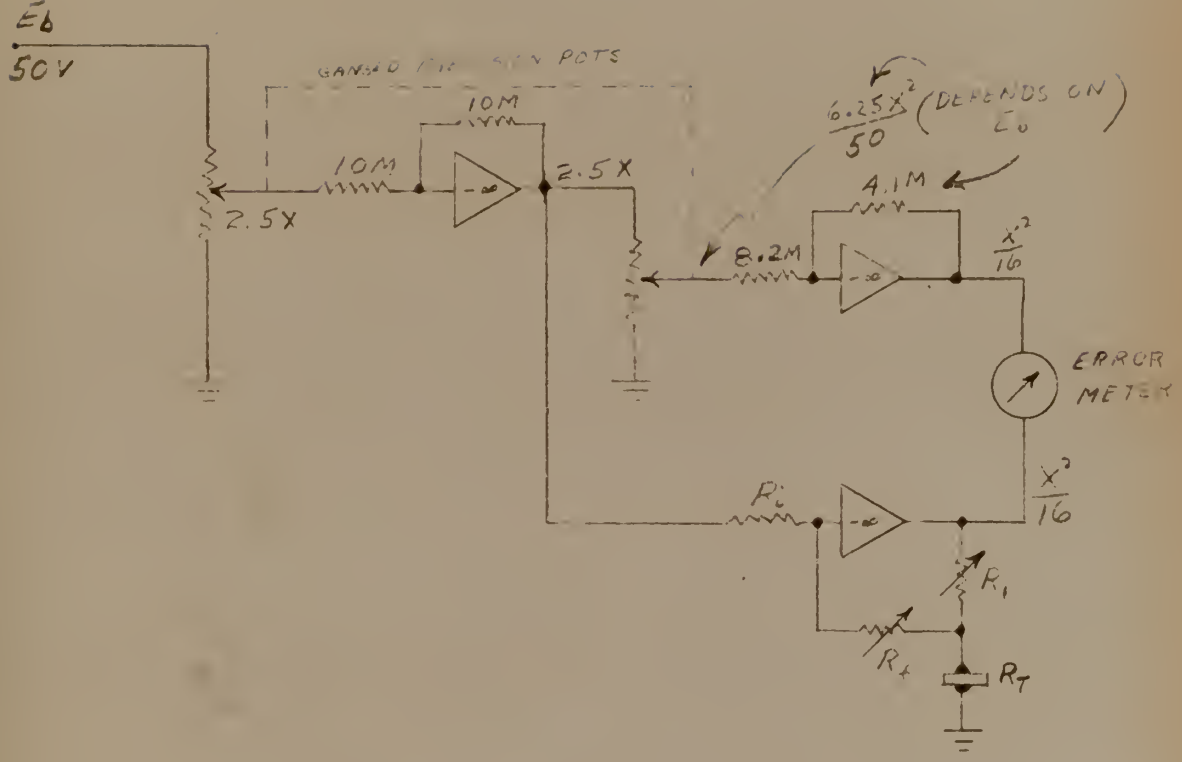
so the output of the second amplifier is

$$\frac{6.25X^2}{100}$$

or

$$\frac{X^2}{16}$$

The input to the experimental squaring circuit is 2.5X, since maximum X is 20 instead of 10. Thus the input really represents 5(A+B) as in the actual circuit, and ranges from zero to fifty volts. The reason this roundabout scaling procedure was used in the test was for simplicity in the perfect squarer for obtaining $\frac{X^2}{16}$ . The input resistor, $R_1$, of the experimental Thyrite squaring circuit is half the value it has in the multiplier since it represents two input resistors for a channel such as




SQUARING ERROR TEST

FIGURE 28

the sum channel with $A = B$. Therefore, the effect of these two input resistors with both input voltages equal is that of a single resistor of half the value with that voltage impressed.

Resistors $R_f$ and $R_1$ are precision decade resistance boxes and $R_t$ is the Thyrite disc. Many combinations of these resistors with three different types of Thyrite (curves shown in Figure 19) were used and the combination that gave the least error on the error meter was picked for the actual multiplier. By turning the shaft of the ganged potentiometers and observing the squaring error on a one volt full scale error meter, the optimum settings for $R_f$ and $R_1$ were rapidly determined. A comparison of the best results from this test with theoretical calculations of the same circuit will be shown in Appendix B. $R_f$ was a one megohm potentiometer to adjust for different pieces of Thyrite of the same general type and to adjust for aging effects, if any.

Test result for:

$R_1 = 18K$

$R_f = 252K$

$R_i = 2.35M$

$$\frac{R_f}{R_i} = .1072$$

| E in | E out | Voltage Error | E Thyrite | $E_T/E_{in}$ |
|---|---|---|---|---|
| 5 | .8 | .55 | .55 | .11 |
| 10 | 1.7 | .70 | 1.2 | .12 |
| 15 | 2.9 | .65 | 1.6 | .107 |
| 20 | 4.5 | .50 | 2.13 | .1065 |
| 25 | 6.5 | .25 | 2.65 | .106 |
| 30 | 9 | 0 | 3.2 | .1068 |
| 35 | 12 | -.25 | 3.7 | .1058 |
| 40 | 16 | 0 | 4.3 | .1075 |
| 45 | 20.2 | 0 | 4.8 | .1068 |

Note that the relation $E_2 = \frac{R_f}{R_i} E_i$ holds true within accuracy of measurements. Six other Thyrites all had errors less that or equal to the error shown here.

## APPENDIX B

## COMPARISON OF THEORETICAL WITH ACTUAL ERRORS IN SQUARING CIRCUITS

Using the theoretical expression for the squaring circuit:

$$E_o = \frac{R_f}{R_i} E_i - \frac{R_1}{R_i} E_i - R_1 I_2 ,$$

where $I_2$ is either picked off the characteristic Thyrite curve for the various values of $E_2$, $(\frac{R_f}{R_i} E_i)$, or found from the theoretical Thyrite expression,

$$I_2 = KE_2^{\,n}$$

The squaring error may be determined.

$R_1 = 18$ Kilohms

$R_f = 165$ Kilohms theoretical (252K in actual circuit)
(This depends on the particular piece of Thyrite that is used)

$R_i = 2.35$ Megohms

$K = 7.2 \times 10^{-5}$

$n = 2.2$

From the chart on the following page it can be seen that there is close correlation between the actual measured errors and the calculated errors.

| E in (volts) | Desired $E_o$ (volts) | Curve $I_2$ Error (volts) | $I_2 = KE_2^n$ (volts) | Measured Error (volts) |
|---|---|---|---|---|
| 5 | .25 | .435 | .46 | .55 |
| 10 | 1.0 | .57 | .50 | .70 |
| 15 | 2.25 | .47 | .45 | .65 |
| 20 | 4.0 | .34 | .35 | .50 |
| 25 | 6.25 | .19 | .15 | .25 |
| 30 | 9.0 | 0 | 0 | 0 |
| 35 | 12.25 | -.4 | -.25 | -.25 |
| 40 | 16.0 | -.3 | -.40 | 0 |
| 45 | 20.25 | -.55 | -.65 | 0 |
| 50 | 25.0 | -.80 | -.8 | |

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